Chapter 1.

1.1

(a) 1

(b) 1

(c) 1

(d) -2

1.2

(a) -1

(b) 3

(c) -3

(d) 2

1.3

(a) ∞

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{4}$

1.4

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

1.5

(a) $y' = 10x^4 + 16x^3 + 8x + 8$

(b) $y' = \dfrac{x+1}{2\sqrt[4]{(x^2+2x+1)^3}}$

(c) $y' = \dfrac{x^2+4x-3}{(x+2)^2}$

(d) $y' = \dfrac{-2x^2+8x+1}{(2x^2+1)^2}$

1.6

(a) $y' = 2(\sin 2x + \cos x)(2\cos 2x - \sin x)$

(b) $y' = -\sin x(2 + \sin 2x) + 2\cos x \cos 2x$

(c) $y' = \dfrac{\sin x - \cos x - 1}{(1 + \cos x)^2}$

(d) $y' = \dfrac{\cos x(\tan x + 1) - \sin x \cdot sec^2 x}{(\tan x + 1)^2}$

(e) $y' = 2^x(\cos x \ln 2 - \sin x)$

(f) $y' = \dfrac{1 - 2x}{e^{2x}}$

(g) $y' = \dfrac{1}{\tan x} \cdot \dfrac{1}{\ln^2} \sec^2 x$

(h) $y' = -\tan x$

(i) $\dfrac{dy}{dx} = 3(2x^2 - x)^2(4x - 1)$

(j) $\dfrac{dy}{dx} = \dfrac{2x}{(x + 1)^3}$

(k) $y' = \dfrac{-1}{3\sqrt[3]{x^4}} + \dfrac{2\cos x(\cos 2x) + 4\sin x \sin 2x}{(\cos 2x)^2}$

(l) $y' = \left(\dfrac{1}{x^3} - \dfrac{2}{x + 2}\right) + x\left(-\dfrac{3}{x^4} + \dfrac{2}{(x + 2)^2}\right)$

(m) $y' = -\dfrac{1}{3\sqrt[3]{x^2}}(\sin\sqrt[3]{x} + \cos\sqrt[3]{x}) - \cos x \sin(\sin x)$

(n) $y' = \dfrac{(1 + 2\ln x)(1 - x - 2x\ln x)}{x^3}$

1.7

(a) $2\ln 2$

(b) ∞

(c) 1

(d) 4

(e) 0

(f) $\dfrac{1}{4}$

1.8

(a) $\dfrac{4}{3}x^{\frac{3}{2}} + \dfrac{3}{4}x^4 + c$

(b) $\dfrac{1}{3}x^3 + \dfrac{1}{2}x^2 + x + c$

(c) $\dfrac{1}{2}x - \dfrac{1}{4}\sin 2x + c$

(d) $\sin x+\cos x+c$

(e) $\tan x-2\sin x+c$

(f) $x-\dfrac{\left(\dfrac{3}{2}\right)^{x}}{\ln\dfrac{3}{2}}+\dfrac{2^{-x}}{\ln 2}+c$

1.9

(a) x^2+3x+c

(b) $\dfrac{4}{3}x^{\frac{3}{2}}+x^3+c$

(c) $-2\cos x+3\sin x+c$

(d) $x+\sin x+c$

(e) $\dfrac{1}{2}\dfrac{4^{2x}}{\ln 4}+3x+c$

(f) $\ln x+2\cot x-3\cos x+c$

1.10

(a) $\dfrac{1}{3}(x^3-1)^3+c$

(b) $\dfrac{1}{9}\left[\dfrac{2}{5}(3x+1)^{5/2}-\dfrac{2}{3}(3x+1)^{3/2}\right]+c$

(c) $\ln(x^2+x-1)+c$

(d) $\dfrac{2}{3}\left[(x+2)^{3/2}+(x-2)^{3/2}\right]+c$

(e) $-\dfrac{1}{6}(\cos 2x)^3+c$

(f) $\dfrac{1}{4}(\sin 2x)^2+c$

(g) $\dfrac{3}{2}(\ln x)^2+c$

(h) $e^{x^2}+c$

(i) $2\sqrt{x^2+x+1}+c$

1.11

(a) $e^x(x-1)+c$

(b) $\dfrac{1}{2}e^x(\sin x-\cos x)+c$

(c) $\dfrac{1}{2}e^x(\sin x-\cos x)-e^x(x-1)+c$

(d) $\dfrac{1}{2}x^2\ln x-\dfrac{1}{4}x^2+c$

1.12

(a) $\frac{29}{6}$

(b) 0

(c) -2

(d) $-\frac{1}{3}(3)^{\frac{3}{2}}-\frac{1}{3}(2)^3+\frac{1}{3}(-1)^{\frac{3}{2}}$

1.13

(a) $\frac{4\sqrt{2}}{3}-\frac{2}{3}$

(b) e^2-e^{-2}

(c) $\frac{1}{3}$

(d) 0

1.14

(a) $-1+\frac{\pi}{12}+\frac{\sqrt{3}}{2}$

(b) -2π

(c) $\frac{1}{4}$

(d) $-\frac{7}{9}+\frac{8}{3}\ln 2$

Chapter 2.

2.1

(a) $y = c\sqrt{x^2-1}+1$

(b) $y = \dfrac{1}{\dfrac{1}{2}x^2+x+c}$

(c) $y = c\dfrac{1}{x}$

(d) $\ln y = e^x + c$

(e) $y = \ln\left(-e^{-x}+x+c\right)$

(f) $y = c\,e^{\frac{1}{2}x^2-2x}$

(g) $y = c\dfrac{1}{x^2}e^x$

(h) $y = ce^{-2\sin x}$

(i) $\dfrac{1}{3}y^3+\dfrac{1}{2}y^2 = x^2+c$

(j) $y = ce^{\frac{1}{2}(\ln x)^2}$

(k) $y^2 = -2\sin x + c$

(l) $y = c\sec x$

(m) $y^2 = -x^2+4$

(n) $y^2 = -x^2+2x$

(o) $y = \dfrac{e^{x^2/2}}{2-e^{x^2/2}}$

(p) $\dfrac{y-1}{y} = \dfrac{1}{2\sqrt{e}}e^{-\frac{1}{2}x^2}$

(q) $y = -e^{-x}-x+2$

(r) $y = \dfrac{2x}{3x-2}$

(s) $y = \dfrac{1}{2}(x+2)$

2.2

$v(1) = 0.2(m/s)$

2.3

$w(t) = w_0 e^{-\frac{c}{J}t}$

2.4

(a) $y = c_1 e^{e^x}$ 또는 $\ln y = e^x + c$

(b) $y = ce^{-x^3}$

(c) $y = c\sec x$

(d) $y = e^{\sin x}$

2.5

(a) $y = -\frac{1}{2} + x + Ce^{-2x}$

(b) $y = \frac{1}{2}(\cos x + \sin x) + ce^{-x}$

(c) $y = c\frac{1}{\sqrt{x^2 - 4}}$

(d) $y = -e^{-x} - \frac{1}{x}e^{-x} + \frac{c}{x}$

(e) $y = -x - 1 + 2e^x$

(f) $y = x[\ln x + 1]$

(g) $y = -x - 3 + 5e^x$

2.6

(a) $y = \frac{3}{3Cx - x^4}$

(b) $y = \frac{1}{\frac{1}{2}e^{-x} + ce^x}$

2.7

(a) $y = \frac{-x + c}{x^2}$

(b) $x^2 y + xy = c$

(c) $\frac{1}{2}x^2 - x + \frac{1}{2}y^2 + y = c$

(d) $2x^2 + xy - 2y^2 = c$

(e) $f(x,y) = 2e^x + 2xy - ye^{-y} - e^{-y} = c$

(f) $f(x,y) = \frac{1}{3}x^3 + y^2 x + y = c$

(g) $\frac{1}{3}x^3 + y^2 x = \frac{4}{3}$ 또는 $y = \sqrt{\frac{4 - x^3}{3x}}$

(h) $y^2 x + 2y = 2$

2.8

(a) $f(x,y) = x^2e^x - xe^x + e^x + y^2e^x = c$ 또는 $y = \pm\sqrt{-1+x-x^2+c\,e^{-x}}$

(b) $f(x,y) = 2xye^{\frac{1}{2}x} = c$

(c) $f(x,y) = -\dfrac{y+1}{x+1} = c$

(d) $f(x,y) = x^2y = c$

(e) $\dfrac{1}{2}x^2y^2 + \dfrac{1}{2}y^4 = c$

(f) $y = \dfrac{c}{\sqrt{x}}$

(g) $y = \dfrac{1}{2}(-2+4x-4x^2+18e^{-2x})^{\frac{1}{2}}$

(h) $y = (4-\ln(x^2+1))^{\frac{1}{2}}$

2.9

$i(0.2) = \dfrac{5}{2} - \dfrac{5}{2}e^{-2(0.2)} \approx 0.82$ [A]

Chapter 3.

3.1

(a) $y = c_1 e^{x} + c_2 e^{-x}$

(b) $y = c_1 e^{2x} + c_2 e^{-2x}$

(c) $y = c_1 e^{2x} + c_2 e^{-x}$

(d) $y = c_1 e^{-x} + c_2 e^{-3x}$

(e) $y = c_1 e^{3x} + c_2 e^{-2x}$

(f) $y = c_1 e^{2x} + c_2 e^{-5x}$

(g) $y = c_1 e^{-x} + c_2 x e^{-x}$

(h) $y = c_1 e^{2x} + c_2 x e^{2x}$

(i) $y = c_1 e^{3x} + c_2 x e^{3x}$

(j) $y = c_1 e^{-4x} + c_2 x e^{-4x}$

(k) $y = c_1 \cos 2x + c_2 \sin 2x$

(l) $y = e^{x}(c_1 \cos \sqrt{2}\,x + c_2 \sin \sqrt{2}\,x)$

(m) $y = e^{x}(c_1 \cos x + c_2 \sin x)$

(n) $y = e^{-x}(c_1 \cos \sqrt{3}\,x + c_2 \sin \sqrt{3}\,x)$

(o) $y = e^{\frac{3}{2}x}(c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x)$

(p) $y = e^{\frac{1}{2}x}(c_1 \cos \frac{\sqrt{11}}{2} x + c_2 \sin \frac{\sqrt{11}}{2} x)$

3.2

(a) $y = \frac{2}{3} e^{3x} + \frac{1}{3} e^{-3x}$

(b) $y = -e^{-x}$

(c) $y = -\frac{1}{2} e^{4x} - \frac{1}{2} e^{-2x}$

(d) $y = -\frac{1}{3} e^{x} + \frac{1}{3} e^{-2x}$

(e) $y = 2e^{2x} - 4x e^{2x}$

(f) $y = e^{-x}(\cos \sqrt{2}\,x + \sqrt{2} \sin \sqrt{2}\,x)$

(g) $y = e^{x}(2\cos 2x - 2\sin 2x)$

(h) $y = e^{\frac{1}{2}x}(-2\cos \frac{\sqrt{15}}{2} x + \frac{4}{\sqrt{15}} \sin \frac{\sqrt{15}}{2} x)$

3.3

(a) $y=c_1e^{3x}+c_2e^{-2x}-\frac{1}{4}e^{-x}$

(b) $y=c_1e^{3x}+c_2xe^{3x}+\frac{1}{9}x^2+\frac{4}{27}x+\frac{2}{27}$

(c) $y=c_1e^{3x}+c_2e^{-3x}-\frac{1}{10}\cos x$

(d) $y=c_1e^{x}+c_2e^{-3x}-\frac{1}{5}\cos x-\frac{2}{5}\sin x$

(e) $y=c_1e^{-x}+c_2e^{-2x}+\frac{1}{6}e^{x}+x-\frac{3}{2}$

(f) $y=c_1e^{x}+c_2e^{-x}+e^{2x}\left(\frac{1}{10}\cos x+\frac{1}{5}\sin x\right)$

(g) $y=c_1e^{x}+c_2e^{-x}+\frac{1}{2}xe^{x}$

(h) $y=c_1e^{2x}+c_2e^{-2x}+\frac{1}{2}xe^{2x}$

(i) $y=c_1e^{-2x}+c_2xe^{-2x}+\frac{1}{2}x^2e^{-2x}$

(j) $y=c_1e^{-x}+c_2xe^{-x}+x^2e^{-x}$

3.4

(a) $y=-\frac{7}{25}e^{x}+\frac{7}{25}e^{-4x}+\frac{2}{5}xe^{x}$

(b) $y=\frac{1}{2}e^{2x}+\frac{10}{17}e^{-3x}-\frac{1}{34}(3\cos x+5\sin x)$

(c) $y=\frac{7}{4}e^{x}+\frac{3}{20}e^{3x}+\frac{1}{10}\cos x-\frac{1}{5}\sin x-e^{-2x}$

(d) $y=\frac{5}{2}e^{-x}-\frac{75}{272}e^{-4x}+\frac{1}{4}x-\frac{5}{16}+\frac{3}{34}\cos x+\frac{5}{34}\sin x$

(e) $y=-\frac{1}{4}e^{2x}-\frac{5}{2}xe^{x}+\frac{1}{2}x^2+x+\frac{5}{4}$

(f) $y=\frac{35}{54}e^{3x}-\frac{14}{9}xe^{3x}+\frac{1}{2}e^{x}+\frac{1}{9}x-\frac{4}{27}$

(g) $y=\frac{31}{45}e^{3x}+\frac{37}{90}e^{-3x}+\frac{1}{6}xe^{3x}-\frac{1}{10}\cos x$

(h) $y=2e^{x}\sin x+1$

(i) $y=\frac{5}{12}e^{2x}-\frac{2}{3}e^{-x}-\frac{1}{2}x+\frac{1}{4}$

(j) $y=\frac{1}{2}e^{-t}+\frac{1}{6}e^{3t}+\frac{1}{3}$

3.5

(a) $y = c_1 e^{x} + c_2 e^{2x} + \frac{1}{2}x + \frac{5}{4}$

(b) $y = c_1 e^{x} + c_2 xe^{x} + x^2 e^{x}$

(c) $y = c_1 e^{x} + c_2 e^{-4x} + \frac{5}{13}e^{x}\sin x - \frac{1}{13}e^{x}\cos x$

(d) $y = c_1 e^{-x} + c_2 e^{2x} - x^2 + x - 1$

(e) $y = c_1 e^{x} + c_2 e^{3x} - e^{2x} + \frac{1}{10}\cos x - \frac{1}{5}\sin x$

(f) $y = c_1 e^{-x} + c_2 e^{-4x} + \frac{1}{4}x - \frac{5}{16} + \frac{1}{34}(3\cos x + 5\sin x)$

(g) $y = c_1 e^{2x} + c_2 xe^{2x} + \left(-\frac{8}{25}x - \frac{44}{125}\right)\sin x + \left(\frac{6}{25}x + \frac{8}{125}\right)\cos x$

(h) $y = c_1 e^{2x} + c_2 xe^{2x} - \frac{1}{32}e^{x} + \frac{1}{16}xe^{x}$

(i) $y = \frac{26}{45}e^{2x} + \frac{1}{5}e^{-2x} - \frac{1}{3}xe^{-x} + \frac{2}{9}e^{-x} - \frac{1}{5}\sin x$

(j) $y = \frac{1}{5}e^{x}(3\sin x - \cos x) - \frac{2}{5}\sin x + \frac{1}{5}\cos x + x + 1$

3.6

(a) $y = c_1 x^{\lambda_1} + c_2 x^{\lambda_2} = c_1 x^{-1} + c_2 x^{-2}$

(b) $y = c_1 x^{\lambda_1} + c_2 x^{\lambda_2} = c_1 x^{-1} + c_2 x^{-1/2}$

(c) $y = x^{-2}(c_1 + c_2 \ln|xvert) = \frac{1}{x^2}(c_1 + c_2 \ln|xvert)$

(d) $y = x^{-1}(c_1 + c_2 \ln|xvert) = \frac{1}{x}(c_1 + c_2 \ln|xvert)$

(e) $y = x^{-1}\left[C_1 \cos(\ln|x|) + C_2 \sin(\ln|x|)\right]$

(f) $y = x^{-\frac{1}{2}}\left[c_1 \cos\left(\frac{1}{2}\ln|xvert\right) + c_2 \sin\left(\frac{1}{2}\ln|xvert\right)\right]$

3.7

(a) $y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{-2x}$

(b) $y = c_1 e^{x} + c_2 e^{-x} + c_3 e^{2x}$

(c) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-2x}$

(d) $y = c_1 e^{-x} + c_2 e^{2x} + c_3 e^{-3x}$

(e) $y = c_1 e^{\frac{1}{2}x} + c_2 e^{-x} + c_3 e^{-2x}$

(f) $y = c_1 e^{x} + c_2 x e^{x} + c_3 e^{-2x}$

(g) $y = c_1 e^{-x} + c_2 x e^{-x} + c_3 e^{2x}$

(h) $y = c_1 e^{x} + e^{-2x}(c_2 + c_3 x)$

(i) $y = (c_1 + c_2 x + c_3 x^2)e^{x} + c_4 e^{-x}$

(j) $y = e^{-x}(c_1 + c_2 x + c_3 x^2) + c_4 e^{2x}$

(k) $y = c_1 e^{-x} + e^{x}(c_2 \cos x + c_3 \sin x)$

(l) $y = c_1 e^{x} + c_2 \cos x + c_3 \sin x$

(m) $y = c_1 \cos x + c_2 \sin x + c_3 \cos 2x + c_4 \sin 2x$

(n) $y = c_1 \cos 2x + c_2 \sin 2x + c_3 x \cos 2x + c_4 x \sin 2x$

(o) $y = c_1 e^{x} + c_2 \cos x + c_3 \sin x + c_4 x \cos x + c_5 x \sin x$

3.8

(a) $x = c_1 e^{-2t} + c_2 e^{-4t}, \quad y = c_1 e^{-2t} - c_2 e^{-4t}$

(b) $x = c_1 e^{2t} + c_2 e^{4t}, \quad y = \frac{1}{3}\left(c_1 e^{2t} + 3c_2 e^{4t}\right)$

(c) $x = c_1 \sin t + c_2 \cos t, \quad y = c_1 \cos t - c_2 \sin t$

(d) $x = (c_1 + c_2 t)e^{t}, \quad y = c_1 e^{t}$

(e) $x = c_1 e^{3t} + c_2 e^{-t}, \quad y = -c_1 e^{3t} + c_2 e^{-t}$

(f) $x = e^{t}(c_1 \sin t + c_2 \cos t), \quad y = -e^{t}(c_1 \cos t - c_2 \sin t)$

(g) $x = c_1 e^{t} - \frac{1}{5} c_2 e^{-4t}, \quad y = c_2 e^{-4t}$

(h) $x = e^{\frac{3}{2}t}(c_1 + c_2 t), \quad y = e^{\frac{3}{2}t}(c_1 + c_2 t - 2c_2)$

(i) $x = c_1 e^{5t} + c_2 e^{-2t} + \frac{1}{5}t - \frac{23}{50}, \quad y = \frac{4}{3}c_1 e^{5t} - c_2 e^{-2t} - \frac{2}{5}t + \frac{11}{50}$

(j) $x = c_1 e^{-5t} + c_2 e^{-2t} + \frac{7}{4}e^{-t}, \quad y = -c_1 e^{-5t} + \frac{1}{2}c_2 e^{-2t} + \frac{5}{4}e^{-t}$

(k) $x = c_1 e^{2t} + c_2 e^{4t} + t + \frac{5}{8}, \quad y = \frac{1}{3}c_1 e^{2t} + c_2 e^{4t} + \frac{1}{8}$

(l) $x = c_1 e^{3t} + c_2 e^{5t} - \frac{61}{130}\cos t + \frac{7}{130}\sin t, \quad y = -c_1 e^{3t} - 2c_2 e^{5t} - \frac{27}{130}\sin t + \frac{31}{130}\cos t$

3.9

(a) $x = e^{t}, \ y = 0$

(b) $x = 3e^{-2t} - e^{-4t}, \ y = 2e^{-2t} - e^{-4t}$

(c) $x = -2\sin t - \cos t, \ y = -2\cos t + \sin t$

(d) $x = e^{t}(2t + 1), \ y = 2e^{t}$

(e) $x = \frac{8}{7}e^{5t} + \frac{13}{7}e^{-2t}, \ y = \frac{6}{7}e^{5t} - \frac{13}{7}e^{-2t}$

(f) $x = e^{-t} - 4e^{-4t}, \ y = 4e^{-t} - 4e^{-4t}$

(g) $x = \frac{2}{5}e^{-4t} - \frac{12}{5}e^{t}, \ y = -2e^{-4t}$

(h) $x = 2te^{2t}, \ y = -e^{2t}(2t - 2)$

(i) $x=\frac{11}{9}e^{-3t}-\frac{29}{18}e^{3t}+\frac{1}{2}e^{t}+\frac{8}{9}$, $y=-\frac{11}{18}e^{-3t}+\frac{29}{9}e^{3t}-\frac{1}{2}e^{t}-\frac{10}{9}$

(j) $x=-\frac{207}{130}e^{5t}+\frac{79}{30}e^{3t}-\frac{14}{15}-\frac{7}{65}\cos t+\frac{4}{65}\sin t$, $y=\frac{207}{65}e^{5t}-\frac{79}{30}e^{3t}-\frac{3}{130}\sin t-\frac{11}{130}\cos t+\frac{8}{15}$

3.10

$x=0.2\cos t$

3.11

$x=-\frac{7}{30}\cos 2t+\frac{1}{10}\sin 2t+\frac{1}{3}\cos t$

3.12

$q=-\frac{5}{2}\cos\sqrt{2}\,t+\frac{5}{2}$

3.13

$x(t)=e^{-t}(0.1\cos t+0.3\sin t)$

3.14

$x(t)=(0.2+0.5t)e^{-2t}$

3.15

$x=0.5e^{-t}-0.2e^{-3t}$

3.16

$q(t)=e^{-t}(-0.2\cos 3t+0.1\sin 3t)+0.5$

Chapter 4.

4.1

(a) 수렴반경 $R=2$

(b) 수렴반경 $R=\infty$

4.2

(a) $y = b_0 + 3b_0 x + \dfrac{3^2 b_0}{2!}x^2 + \dfrac{3^3 b_0}{3!}x^3 + \cdots = b_0\left(1 + 3x + \dfrac{1}{2!}(3x)^2 + \dfrac{1}{3!}(3x)^3 + \cdots\right) = b_0 e^{3x}$

(b) $y = b_0 - b_0 x + \dfrac{b_0}{2!}x^2 - \dfrac{b_0}{3!}x^3 + \cdots = b_0 e^{-x}$

(c) $y = ce^{-x} - 2 + 2x$

(d) $y = b_0 + b_1 x + \dfrac{1}{2!}b_1 x^2 + \dfrac{1}{3!}b_1 x^3 + \cdots = b_0 - b_1 + (b_1 + b_1 x + \dfrac{1}{2!}b_1 x^2 + \dfrac{1}{3!}b_1 x^3 + \cdots) = b + b_1 e^x$

(e) $y = c_1 e^{2x} + c_2 e^{-2x}$

(f) $y = b_0 \cos 3x + b \sin 3x$

(g) $y = c_1 \cos 2x + c_2 \sin 2x$

4.3

(a) $y = e^{2x}$

(b) $y = e^{-3x}$

(c) $y = e^{x}$

(d) $y = \dfrac{1}{2}e^{2x} + \dfrac{1}{2}e^{-2x}$

4.4

(a) $y = b_0 e^{(1/2)x^2}$

(b) $y = c_1\left(1 - \dfrac{1}{2}x^2 - \dfrac{1}{24}x^4 - \dfrac{1}{240}x^6 - \cdots\right) + c_2 x$

(c) $y = b_0\left(1 - \dfrac{1}{2}x^2 + \dfrac{1}{8}x^4 - \dfrac{1}{48}x^6 + \cdots\right) + b_1\left(x - \dfrac{1}{3}x^3 + \dfrac{1}{15}x^5 - \cdots\right)$

(d) $y = c_1\left(1 + x^2 + \dfrac{1}{2}x^4 + \dfrac{1}{6}x^6 + \cdots\right) + c_2\left(x + \dfrac{2}{3}x^3 + \dfrac{4}{15}x^5 + \dfrac{8}{105}x^7 + \cdots\right)$

(e) $y = c_1 y_1 + c_2 y_2 = c_1\left(1 + \dfrac{1}{6}x^3 + \dfrac{1}{180}x^6 + \cdots\right) + c_2\left(x + \dfrac{1}{12}x^4 + \dfrac{1}{504}x^7 + \cdots\right)$

(f) $b_2 = 0$, $b_k = -\dfrac{b_{k-3}}{k(k-1)}$, $k = 3, 4, 5, \cdots$

$b_0 = 1$, $b_1 = 0$이라 놓으면 $b_3 = -\dfrac{1}{6}$, $b_4 = b_5 = 0$, $b_6 = \dfrac{1}{180}$

$b_0 = 0$, $b_1 = 1$이라 놓으면 $b_3 = 0$, $b_4 = -\dfrac{1}{12}$, $b_5 = b_6 = 0$, $b_7 = \dfrac{1}{504}$, $\cdots$

$\therefore y_1 = b_0(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \cdots)$, $y_2 = b_1(x - \frac{1}{12}x^4 + \frac{1}{504}x^7 - \cdots)$

$y = b_0 y_1 + b_1 y_2$ 초깃값을 대입하여 b_0와 b_1을 구하면 된다.

(g) $y = x + \frac{1}{3}x^3 + \frac{1}{15}x^5 + \frac{1}{105}x^7 + \cdots$

4.5

(a) $y = C_1 x + C_2 x^{1/3}$

(b) $y = y_1 + y_2 = c_1 x + \frac{c_2}{x^2}$

(c) $y = c(1 + \frac{1}{4}x^2 + \frac{1}{64}x^4 + \cdots)$

(d) $y = c_1(1 + 2x - 2x^2 + \frac{4}{9}x^3 + \cdots) + c_2 x^{\frac{3}{2}}(1 - \frac{2}{5}x + \frac{2}{35}x^2 - \frac{4}{945}x^3 + \cdots)$

(e) $y = C_1 e^x + C_2 e^x \ln|x|$

(f) $y = \frac{1}{\sqrt{x}}(C_1 \sin x + C_2 \cos x)$

(g) $y_1 = b_o x(1 - \frac{1}{4}x^2 + \frac{1}{64}x^4 - \frac{1}{2304}x^6 + \cdots)$

$$y_2 = y_1 \int \frac{e^{\frac{dx}{x}}}{y_1^2}dx = y_1 \int \frac{1}{x(1 - \frac{1}{2}x^2 + \frac{3}{32}x^4 - \frac{5}{576}x^6 + \cdots)}dx$$

$$= y_1 \ln x + y_1(\frac{1}{4}x^2 + \frac{5}{128}x^4 + \frac{2x}{3456}x^6 + \cdots)$$

$y = c_1 y_1 + c_2 y_2$

(h) $y = c_1(1 + \frac{1}{2}x + \frac{1}{24}x^2 + \frac{1}{720}x^3 + \cdots) + c_2\sqrt{x}(1 + \frac{1}{6}x + \frac{1}{120}x^2 + \frac{1}{5040}x^3 + \cdots)$

4.6

$y = c_1(1 - 3x^2) + c_2(x - \frac{1}{2}x^3 - \frac{24}{5!}x^5 - \cdots)$

4.7

(a) $y = c_1 J_{1/4}(x) + c_2 Y_{1/4}(x)$

(b) $y = c_1 J_0(4\sqrt{x}) + c_2 Y_0(4\sqrt{x})$

(c) $y = c_1 J_2(3x) + c_2 Y_2(3x)$

Chapter 5.

5.1

(a) $\mathcal{L}\{5t^2+2\}=5\frac{2}{s^3}+2\frac{1}{5}$

(b) $\mathcal{L}\{(2t+1)^2\}=4\frac{2}{s^3}+4\frac{1}{s^2}+\frac{1}{s}$

(c) $\mathcal{L}\{\cos 2t\}=\frac{s}{s^2+4}$

(d) $\mathcal{L}\{\sin 3t\}=\frac{3}{s^2+9}$

(e) $\mathcal{L}\{e^{2t-2}\}=\frac{e^{-2}}{s-2}$

(f) $\mathcal{L}\{te^{-2t}\}=\frac{1}{(s+2)^2}$

(g) $\mathcal{L}\{t^2e^{2t}\}=\frac{2}{(s-2)^3}$

(h) $\mathcal{L}\{e^{-2t}\cos 2t\}=\frac{(s+2)}{(s+2)^2+2^2}$

(i) $\mathcal{L}\{2t\sin t\}=\frac{4s}{(s^2+1)^2}$

(j) $\mathcal{L}\{t\cos 2t\}=\frac{s^2-2^2}{(s^2+2^2)^2}$

(k) $\mathcal{L}\{\sin 4t\cos 4t\}=\frac{4}{s^2+64}$

(l) $\mathcal{L}\{\cos^2 at\}=\frac{1}{2s}+\frac{s}{2(s^2+4a^2)}$

(m) $\mathcal{L}\{f(t)\}=-\frac{4}{s}e^{-2s}+\frac{2}{s}$

(n) $\mathcal{L}\{f(t)\}=-\frac{1}{s}[e^{-s}-1]$

(o) $\mathcal{L}\{f(t)\}=\frac{1}{s+1}[1-e^{-(s+1)}]$

(p) $\mathcal{L}\{f(t)\}=\frac{2}{-(s+1)}[e^{-(s+1)}-1]$

(q) $\mathcal{L}\{f(t)\}=-\frac{2}{s}[e^{-s}-1]$

(r) $\mathcal{L}\{f(t)\}=-\frac{1}{s}[e^{-2s}-e^{-s}]$

(s) $\mathcal{L}\{f(t)\}=\frac{e^{-s}}{s^2}$

(t) $\mathcal{L}\{f(t)\}=\frac{e^{-s}}{s}+\frac{1}{s^2}[e^{-2s}-e^{-s}]$

5.2

(a) $\mathcal{L}\,[f(t)] = \dfrac{1}{1-e^{-2s}}\left(\dfrac{1}{s} - 2\dfrac{e^{-s}}{s} + \dfrac{e^{-2s}}{s}\right)$

(b) $\mathcal{L}\,\{f(t)\} = \dfrac{1}{1-e^{-2s}}\left[\dfrac{2}{s}e^{-s} - \dfrac{1}{s} - \dfrac{1}{s}e^{-2s}\right]$

(c) $\mathcal{L}\,\{f(t)\} = \dfrac{1}{1-e^{-3s}}\left[-\dfrac{2}{s}e^{-s} + \dfrac{1}{s} + \dfrac{1}{s}e^{-2s}\right]$

(d) $\mathcal{L}\,\{f(t)\} = \dfrac{1}{1-e^{-2s}}\left\{-\dfrac{1}{s}e^{-s} - \dfrac{1}{s^2}\left[e^{-s} - 1\right]\right\}$

5.3

(a) $\mathcal{L}\,\left[e^{-2t}t^2\right] = \dfrac{2}{(s+2)^3}$

(b) $\mathcal{L}\,\{f(t)\} = \dfrac{s+3}{(s+1)^2 + 2^2}$

(c) $\mathcal{L}\,\left[e^{t}\sin 2t\right] = \dfrac{2}{(s-1)^2 + 2^2}$

(d) $\mathcal{L}\,\left\{e^{-2t}\sin(2t+\theta)\right\} = \dfrac{2\cos\theta + (s+2)\sin\theta}{(s+2)^2 + 2^2}$

(e) $\mathcal{L}\,\{(t-2)u(t-2)\} = e^{-2s}\dfrac{1}{s^2}$

(f) $\mathcal{L}\,\{2t\cdot u(t-1)\} = 2e^{-s}\dfrac{1}{s^2} + 2\dfrac{e^{-s}}{s}$

(g) $\mathcal{L}\,\left\{e^{2t}u(t-1)\right\} = \dfrac{e^{-(s-2)}}{s-2}$

(h) $\mathcal{L}\,\{u(t-\pi)\cos 2t\} = e^{-\pi s}\dfrac{s}{s^2 + 2^2}$

5.4

(a) $\mathcal{L}\,[f(t)] = \dfrac{1}{(s+2)^2}$

(b) $\mathcal{L}\,[f(t)] = \dfrac{2}{(s-2)^3}$

(c) $\mathcal{L}\,[f(t)] = \dfrac{s^2-4}{(s^2+4)^2}$

(d) $\mathcal{L}\,[f(t)] = \dfrac{4(3s^2-4)}{(s^2+4)^3}$

(e) $\mathcal{L}\,[f(t)] = \dfrac{4(s+1)}{[(s+1)^2+2^2]^2}$

(f) $\mathcal{L}\,[f(t)] = \dfrac{s^2-2s-3}{[(s-1)^2+2^2]^2}$

5.5

(a) $\mathcal{L}[f(t)] = \dfrac{4}{s^4}$

(b) $\mathcal{L}[f(t)] = \dfrac{1}{s^2-1}$

(c) $\mathcal{L}[f(t)] = \dfrac{2}{s^3-2s^2}$

(d) $\mathcal{L}[f(t)] = \dfrac{s}{(s^2+1)^2}$

(e) $\mathcal{L}[f(t)] = \dfrac{s^2}{(s^2+1)(s^2+4)}$

(f) $\mathcal{L}[f(t)] = \dfrac{1}{s^2+1}$

5.6

(a) $\mathcal{L}\left[\int_0^t e^{-2\tau}d\tau\right] = \dfrac{1}{s(s+2)}$

(b) $\mathcal{L}\left[\int_0^t \sin 2\tau d\tau\right] = \dfrac{2}{s^3+4s}$

5.7

(a) $f(t) = u(t) - \dfrac{1}{3}e^{t} - \dfrac{2}{3}e^{-2t}$

(b) $\mathcal{L}^{-1}[F(s)] = f(t) = e^{2t}$

(c) $\mathcal{L}^{-1}[F(s)] = \cos 2t + \dfrac{3}{2}\sin 2t$

(d) $\mathcal{L}^{-1}[F(s)] = u(t) + 2t$

(e) $f(t) = -\dfrac{1}{4}e^{t} + \dfrac{3}{2}te^{t} + \dfrac{1}{4}e^{-t}$

(f) $\mathcal{L}^{-1}[F(s)] = 2u(t) - t + e^{3t}$

(g) $\mathcal{L}^{-1}[F(s)] = -\dfrac{2}{9}u(t) + \dfrac{1}{9}e^{3t} + \dfrac{1}{9}e^{-3t}$

(h) $\mathcal{L}^{-1}[F(s)] = -\dfrac{1}{3}e^{-2t} + \dfrac{1}{3}e^{t}$

(i) $\mathcal{L}^{-1}[F(s)] = \dfrac{1}{2}e^{t} - \dfrac{1}{2}(\cos t + \sin t)$

(j) $\mathcal{L}^{-1}[F(s)] = \cos 2t + \dfrac{1}{2}\sin 2t$

(k) $\mathcal{L}^{-1}[F(s)] = -\dfrac{1}{20}e^{-2t} + \dfrac{1}{20}e^{2t} - \dfrac{1}{5}\sin t$

(l) $\mathcal{L}^{-1}[F(s)] = -\dfrac{3}{2}u(t) + \dfrac{1}{6}e^{-2t} + \dfrac{4}{3}e^{t}$

(m) $\mathcal{L}^{-1}[F(s)] = -\dfrac{5}{2}e^{t} + \dfrac{1}{6}e^{-t} + \dfrac{7}{3}e^{2t}$

(n) $\mathcal{L}^{-1}[F(s)] = -\frac{1}{2}u(t) + \frac{1}{10}e^{-2t} + \frac{3}{20}e^{3t} + \frac{1}{4}e^{-t}$

(o) $\mathcal{L}^{-1}[F(s)] = \frac{3}{32}e^{2t} + \frac{1}{32}e^{-2t} - \frac{1}{16}\sin 2t - \frac{1}{8}\cos 2t$

5.8

(a) $f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{2}t^2 e^{-t}$

(b) $f(t) = 2e^{2t}\cos t + 7e^{2t}\sin t$

(c) $f(t) = e^{-2t}\cos 2t - e^{-2t}\sin 2t$

(d) $f(t) = e^{-t}\cos 2t + e^{-t}\sin 2t$

(e) $f(t) = 2e^{-3t}\cos 2t - \frac{5}{2}e^{-3t}\sin 2t$

(f) $f(t) = 3e^{-2t} - 5te^{-2t}$

(g) $f(t) = 2t - 3u(t) + te^{-t} + 3e^{-t}$

(h) $f(t) = (t-1)u(t-1)$

(i) $f(t) = 2e^{-t} + e^{-(t-2)}u(t-2)$

(j) $f(t) = \frac{1}{2}\sin 2(t-2)u(t-2)$

(k) $f(t) = \frac{1}{4}e^{t+2} - \frac{1}{4}e^{-3(t+2)}$

(l) $f(t) = \frac{1}{3}e^{-(t-1)}u(t-1) + \frac{2}{3}e^{2(t-1)}u(t-1)$

(m) $f(t) = -\frac{1}{5}e^{t-2}u(t-2) + \frac{1}{5}\cos 2(t-2)u(t-2) + \frac{3}{5}\sin 2(t-2)u(t-2)$

5.9

(a) $y(t) = e^{-2t}$

(b) $y(t) = 2e^{-\frac{1}{2}t}$

(c) $y(t) = 3e^{2t} - e^{t}$

(d) $y(t) = \frac{2}{5}e^{2t} + \frac{1}{5}\sin t - \frac{2}{5}\cos t$

(e) $y(t) = e^{t}$

(f) $y(t) = -\frac{3}{2} + \frac{1}{4}(1+\sqrt{2})e^{\sqrt{2}t} + \frac{1}{4}(1-\sqrt{2})e^{-\sqrt{2}t}$

(g) $y(t) = \frac{1}{8}e^{-2t} + \frac{1}{8}\sin 2t - \frac{1}{8}\cos 2t$

(h) $y(t) = \frac{1}{6}e^{-t} + \frac{3}{2}e^{t} - \frac{2}{3}e^{2t}$

(i) $y(t) = -\frac{2}{27}e^{t} + \frac{1}{9}te^{t} + \frac{2}{27}e^{-2t} + \frac{1}{9}te^{-2t}$

(j) $y(t) = -\frac{3}{2}u(t) - t + \frac{5}{3}e^{t} - \frac{1}{6}e^{-2t}$

5.10

(a) $y(t)=e^{2t}+te^{2t}$

(b) $y(t)=te^{-t}+e^{-t}$

(c) $y(t)=-e^{t}\cos 2t+e^{t}\sin 2t$

(d) $y(t)=-\frac{1}{4}u(t)+\frac{1}{4}t+\frac{1}{4}e^{-2t}+\frac{1}{4}te^{-2t}$

(e) $y(t)=\frac{1}{4}e^{-t}-\frac{1}{4}e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t$

(f) $y(t)=e^{-t}-e^{-2t}-te^{-2t}$

(g) $y(t)=\frac{1}{5}\sin t-\frac{2}{5}\cos t+\frac{1}{5}e^{-t}\sin t+\frac{2}{5}e^{-t}\cos t$

5.11

(a) $y(t)=-2u(t-2)+2e^{t-2}u(t-2)$

(b) $y(t)=-\frac{1}{9}u(t)+\frac{1}{3}t+\frac{1}{9}e^{-3t}+\frac{1}{9}u(t-2)-\frac{1}{3}(t-2)u(t-2)+\frac{5}{9}e^{-3(t-2)}u(t-2)$

(c) $y(t)=-u(t)+\cos t+u(t-2)-\cos(t-2)u(t-2)$

(d) $y(t)=\frac{1}{3}e^{-t}+\frac{2}{3}e^{2t}-\frac{1}{2}u(t-1)+\frac{1}{3}e^{-(t-1)}u(t-1)+\frac{1}{6}e^{2(t-1)}u(t-1)$

(e) $y(t)=\frac{1}{5}e^{2t}-\frac{1}{5}e^{-3t}-\frac{1}{6}u(t-1)+\frac{1}{10}e^{2(t-1)}u(t-1)+\frac{1}{15}e^{-3(t-1)}u(t-1)$

$\quad +\frac{1}{6}u(t-2)-\frac{1}{10}e^{2(t-2)}u(t-2)-\frac{1}{15}e^{-3(t-2)}u(t-2)$

(f) $y(t)=\sin t+\frac{1}{2}(t-\pi)\sin(t-\pi)u(t-\pi)$

5.12

(a) $y=-\frac{3}{25}\cos t-\frac{4}{25}\sin t+\frac{2}{5}t\cos t+\frac{1}{5}t\sin t+\frac{28}{25}e^{-2t}$

(b) $y=-\frac{1}{75}e^{-2t}+\frac{1}{12}e^{t}+\frac{1}{100}e^{3t}(10t-7)$

5.13

$x(t)=\frac{3}{2}u(t)-\frac{3}{2}\cos\sqrt{2}\,t-\frac{3}{2}u(t-2)+\frac{3}{2}\cos\sqrt{2}(t-2)u(t-2)$

5.14

$q(t)=\frac{1}{4}u(t-1)-\frac{1}{4}e^{-2(t-1)}u(t-1)$

5.15

$q(t)=\frac{5}{2}u(t-2)-\frac{5}{2}e^{-2(t-2)}u(t-2)+e^{-2t}$

5.16

(a) $x(t)=\cos t+\sin t,\quad y(t)=\cos t-\sin t$

(b) $x(t)=e^{3t},\quad y(t)=e^{3t}$

(c) $x(t)=2\cos\sqrt{3}\,t,\quad y(t)=\cos\sqrt{3}\,t+\sqrt{3}\sin\sqrt{3}\,t$

(d) $x(t)=-\frac{2}{13}e^{-2t}+\frac{2}{13}e^{t}(\cos 2t+5\sin 2t)$

$y(t)=-\frac{7}{13}e^{-2t}+\frac{4}{13}e^{t}(5\cos 2t-\sin 2t)$

(e) $x(t)=-\frac{3}{10}e^{-t}+\frac{3}{10}e^{t}-\frac{2}{15}\sin 2t-\frac{1}{3}\sin t$

$y(t)=\frac{3}{10}e^{-t}-\frac{3}{10}e^{t}+\frac{7}{15}\sin 2t-\frac{1}{2}\sin t$

(f) $x(t)=\frac{1}{4}e^{t}-\frac{2}{7}e^{-2t}+\frac{1}{7}e^{2t}-\frac{\sqrt{3}}{28}\sin\sqrt{3}\,t-\frac{3}{28}\cos\sqrt{3}\,t$

$y(t)=-\frac{2}{7}e^{-2t}+\frac{1}{7}e^{2t}+\frac{\sqrt{3}}{21}\sin\sqrt{3}\,t+\frac{1}{7}\cos\sqrt{3}\,t$

Charter 6.

6.1

(a) $2\vec{a}-\vec{b} = \langle 4,\ 3,\ -4 \rangle$

(b) $-\vec{a}-\vec{b}+2\vec{c} = -\langle 4,\ -5,\ -3 \rangle$

(c) $2\vec{a}+\vec{b}-2\vec{c} = \langle -2, 7, 0 \rangle$

(d) $\| \vec{a}-\overrightarrow{2b} \| = \sqrt{2^2+0^2+1^2} = \sqrt{5}$

(e) $\| \vec{a}-\overrightarrow{2b}-2\vec{c} \| = \sqrt{74}$

(f) $\| -\vec{a}+\vec{b}-2\vec{c} \| = \sqrt{146}$

6.2

(a) $\| \overrightarrow{p_1p_2} \| = \sqrt{2}$

(b) $\| \overrightarrow{p_1p_2} \| = \sqrt{20}$

(c) $\| \overrightarrow{p_1p_2} \| = 5$

(d) $\| \overrightarrow{p_1p_2} \| = \sqrt{45}$

(e) $\| \overrightarrow{p_1p_2} \| = \sqrt{3}$

(f) $\| \overrightarrow{p_1p_2} \| = \sqrt{19}$

(g) $\| \overrightarrow{p_1p_2} \| = \sqrt{11}$

(h) $\| \overrightarrow{p_1p_2} \| = \sqrt{30}$

6.3

(a) $\vec{a} \bullet \vec{b} = 7$

(b) $\vec{a} \bullet \vec{b} = -2$

(c) $\vec{a} \bullet \vec{b} = -9$

(d) $\vec{a} \bullet \vec{b} = 10$

(e) $\vec{a} \bullet \vec{b} = 3$

(f) $\vec{a} \bullet \vec{b} = -2$

(g) $\vec{a} \bullet \vec{b} = 10$

(h) $\vec{a} \bullet \vec{b} = 3$

6.4

(a) $\theta = \cos^{-1}(\frac{1}{\sqrt{5} \bullet \sqrt{13}}) = 82.9^\circ$

(b) $\theta = \cos^{-1}(\frac{-2}{\sqrt{8} \bullet \sqrt{5}}) = 108.4^\circ$

(c) $\theta = \cos^{-1}(0) = 90^\circ$

(d) $\theta = \cos^{-1}(0) = 90^\circ$

(e) $\theta = \cos^{-1}(\frac{5}{\sqrt{5} \bullet \sqrt{6}}) = 24.1^\circ$

(f) $\theta = \cos^{-1}(\frac{2}{3 \bullet \sqrt{22}}) = 81.8^\circ$

(g) $\theta = \cos^{-1}(\frac{-1}{\sqrt{10} \bullet \sqrt{21}}) = 94.0^\circ$

(h) $\theta = \cos^{-1}(0) = 90^\circ$

6.5

(a) $c = -2$

(b) $c = -2$

(c) $c = 5$

(d) $c = \frac{4}{5}$

6.6

(a) $W = -8\,(N \bullet m)$

(b) $W = 10\,(N \bullet m)$

(c) $W = -1\,(N \bullet m)$

6.7

(a) 방향코사인

$\cos\alpha = \frac{3}{\sqrt{14}}$, $\cos\beta = \frac{2}{\sqrt{14}}$, $\cos\gamma = \frac{1}{\sqrt{14}}$

방향각

$\alpha = \cos^{-1}(\frac{3}{\sqrt{14}}) = 36.7^\circ$

$\beta = \cos^{-1}(\frac{2}{\sqrt{14}}) = 57.7^\circ$

$\gamma = \cos^{-1}(\frac{1}{\sqrt{14}}) = 74.5^\circ$

(b) 방향코사인

$\cos\alpha = \frac{3}{\sqrt{14}}$, $\cos\beta = \frac{-1}{\sqrt{14}}$, $\cos\gamma = \frac{2}{\sqrt{14}}$

방향각

$\alpha = \cos^{-1}(\frac{3}{\sqrt{14}}) = 36.7^\circ$

$\beta = \cos^{-1}(\frac{-1}{\sqrt{14}}) = 105.5^\circ$

$\gamma = \cos^{-1}(\frac{2}{\sqrt{14}}) = 57.7^\circ$

(c) 방향코사인

$\cos\alpha = \dfrac{-2}{3}$, $\cos\beta = \dfrac{-2}{3}$, $\cos\gamma = \dfrac{1}{3}$

방향각

$\alpha = \cos^{-1}(\dfrac{-2}{3}) = 131.8^{\circ}$

$\beta = \cos^{-1}(\dfrac{-2}{3}) = 131.8^{\circ}$

$\gamma = \cos^{-1}(\dfrac{1}{3}) = 70.5^{\circ}$

(d) 방향코사인

$\cos\alpha = \dfrac{2}{\sqrt{17}}$, $\cos\beta = \dfrac{-3}{\sqrt{17}}$, $\cos\gamma = \dfrac{-2}{\sqrt{17}}$

방향각

$\alpha = \cos^{-1}(\dfrac{2}{\sqrt{17}}) = 61.0^{\circ}$

$\beta = \cos^{-1}(\dfrac{-3}{\sqrt{17}}) = 136.7^{\circ}$

$\gamma = \cos^{-1}(\dfrac{-2}{\sqrt{17}}) = 119.0^{\circ}$

(e) 방향코사인

$\cos\alpha = \dfrac{-3}{\sqrt{29}}$, $\cos\beta = \dfrac{4}{\sqrt{29}}$, $\cos\gamma = \dfrac{-2}{\sqrt{29}}$

방향각

$\alpha = \cos^{-1}(\dfrac{-3}{\sqrt{29}}) = 123.9^{\circ}$

$\beta = \cos^{-1}(\dfrac{4}{\sqrt{29}}) = 42.0^{\circ}$

$\gamma = \cos^{-1}(\dfrac{-2}{\sqrt{29}}) = 111.8^{\circ}$

(f) 방향코사인

$\cos\alpha = \dfrac{2}{\sqrt{6}}$, $\cos\beta = \dfrac{1}{\sqrt{6}}$, $\cos\gamma = \dfrac{1}{\sqrt{6}}$

방향각

$\alpha = \cos^{-1}(\dfrac{2}{\sqrt{6}}) = 35.3^{\circ}$

$\beta = \cos^{-1}(\dfrac{1}{\sqrt{6}}) = 65.9^{\circ}$

$\gamma = \cos^{-1}(\dfrac{1}{\sqrt{6}}) = 65.9^{\circ}$

6.8

방향코사인

$$\cos\alpha_S = \frac{8}{\sqrt{180}}, \cos\beta_E = \frac{10}{\sqrt{180}}, \cos\gamma_N = \frac{4}{\sqrt{180}}$$

방향각

$$\alpha_S = \cos^{-1}\left(\frac{8}{\sqrt{180}}\right) = 53.4^\circ$$

$$\beta_E = \cos^{-1}\left(\frac{10}{\sqrt{180}}\right) = 41.8^\circ$$

$$\gamma_N = \cos^{-1}\left(\frac{4}{\sqrt{180}}\right) = 72.7^\circ$$

6.9

(a) $comp_{\vec{b}}\vec{a} = \dfrac{-4}{\sqrt{5}}$

(b) $comp_{\vec{b}}\vec{a} = \dfrac{8}{\sqrt{14}}$

(c) $comp_{\vec{b}}\vec{a} = \dfrac{4}{\sqrt{11}}$

6.10

(a) $proj_{\vec{b}}\vec{a} = \left\langle -\dfrac{1}{3}, -\dfrac{1}{3}, \dfrac{2}{3} \right\rangle$

(b) $proj_{\vec{b}}\vec{a} = \left\langle \dfrac{14}{6}, -\dfrac{7}{6}, \dfrac{7}{6} \right\rangle$

(c) $proj_{\vec{b}}\vec{a} = \left\langle -\dfrac{4}{11}, -\dfrac{4}{11}, \dfrac{12}{11} \right\rangle$

(d) $proj_{\vec{b}}\vec{a} = \left\langle \dfrac{1}{3}, \dfrac{1}{3}, -\dfrac{2}{3} \right\rangle$

6.11

(a) $proj_{\vec{b}}\vec{a} = \left\langle \dfrac{8}{9}, \dfrac{16}{9}, \dfrac{-16}{9} \right\rangle$

(b) $proj_{\overrightarrow{b^{\perp}}}\vec{a} = \left\langle \dfrac{10}{9}, \dfrac{2}{9}, \dfrac{7}{9} \right\rangle$

(c) $proj_{(\vec{a}+\vec{b})}\vec{a} = \left\langle \dfrac{3}{2}, 2, -\dfrac{3}{2} \right\rangle$

(d) $proj_{(\vec{a}-\vec{b})}\vec{a} = \left\langle \dfrac{1}{2}, 0, \dfrac{1}{2} \right\rangle$

6.12

(a) $\vec{a}\times\vec{b} = 7\vec{k}$

(b) $\vec{a}\times\vec{b} = \vec{k}$

(c) $\vec{a}\times\vec{b} = 11\vec{k}$

(d) $\vec{a}\times\vec{b} = -7\vec{k}$

(e) $\vec{a}\times\vec{b} = 8\vec{i} - 7\vec{j} + 2\vec{k}$

(f) $\vec{a}\times\vec{b} = -6\vec{i} - 3\vec{k}$

(g) $\vec{a}\times\vec{b} = -9\vec{i} - 6\vec{j} - 6\vec{k}$

(h) $\vec{a}\times\vec{b} = -10\vec{i} + 6\vec{j} + 4\vec{k}$

6.13

(a) $\theta = \sin^{-1}\left(\frac{8}{\sqrt{5}\cdot\sqrt{13}}\right) = 82.9^{\circ}$

(b) $\theta = \sin^{-1}\left(\frac{6}{\sqrt{8}\cdot\sqrt{5}}\right) = 71.6^{\circ}$

(c) $\theta = \sin^{-1}(1) = 90^{\circ}$

(d) $\theta = \sin^{-1}(\frac{9}{\sqrt{18}\cdot\sqrt{5}}) = 71.6^{\circ}$

(e) $\theta = \sin^{-1}(\frac{3}{\sqrt{5}\cdot\sqrt{2}}) = 71.6^{\circ}$

(f) $\theta = \sin^{-1}(\frac{\sqrt{35}}{\sqrt{14}\cdot\sqrt{6}}) = 40.2^{\circ}$

(g) $\theta = \sin^{-1}\left(\frac{\sqrt{11}}{6}\right) = 33.6^{\circ}$

(h) $\theta = \sin^{-1}(\frac{\sqrt{83}}{\sqrt{14}\cdot\sqrt{6}}) = 83.7^{\circ}$

6.14

(a) $\triangle P_1P_2P_3$의 면적 $= \frac{\sqrt{206}}{2} = 7.176$

(b) $\triangle P_1P_2P_3$의 면적 $= \frac{3\sqrt{5}}{2}$

6.15

(a) 평행사변형의 면적은 $\|\vec{a}\times\vec{b}\| = 26.3$

(b) 평행사변형의 면적은 $\|\vec{a}\times\vec{b}\| = \sqrt{142}$

6.16

$M = \overrightarrow{F} \times \overrightarrow{r} = 1000(N \cdot m)$

6.17

(a) 23

(b) 6

6.18

(a) 주어진 세 벡터는 동일 평면상에 있지 않다.

(b) 주어진 세 벡터는 동일 평면상에 있다.

6.19

(a) 세 벡터는 일차독립이다.

(b) 세 벡터는 일차독립이다.

(c) 세 벡터는 일차종속이다.

6.20

(a) $\overrightarrow{a} \cdot \overrightarrow{b} = -15$

(b) $\overrightarrow{a} \cdot \overrightarrow{b} = 2$

6.21

(a) 두 벡터는 서로 직교한다.

(b) 두 벡터는 서로 직교한다.

Charter 7.

7.1

(a) $x = \cos y$

그래프는 코사인 그래프

(b) $\vec{r}(t) = t\vec{i} + 2t^2\vec{j}$, $y = 2x^2$

그래프는 포물선

(c) $x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2 = 2^2$

그래프는 중심이 원점이고, 반경이 2인 원

(d) $x^2 + \dfrac{y^2}{2^2} = 1$

그래프는 중심이 원점이고, 장축이 2, 단축이 1인 타원

(e) $x^2 + y^2 = 9$, $z = 2$

그래프는 $z = 2$인 xy평면상에서 중심이 원점이고, 반경이 3인 원

(f) $x = 2$, $y^2 + z^2 = 1$

그래프는 $x = 2$인 yz평면상에서 중심이 원점이고, 반경이 1인 원

7.2

(a) $\lim_{t \to 0} \vec{r}(t) = \vec{i}$

(b) $\lim_{t \to 0} \vec{r}(t) = -\vec{j}$

(c) $\lim_{t \to 0} \vec{r}(t) = \vec{i} + 2\vec{j} - \dfrac{1}{2}\vec{k}$

(d) $\lim_{t \to 0} \vec{r}(t) = -\vec{j} + \vec{k}$

7.3

(a) $\lim_{t \to \infty} \vec{r}(t) = 2\vec{j}$

(b) $\lim_{t \to \infty} \vec{r}(t) = 2\vec{i}$

(c) $\lim_{t \to \infty} \vec{r}(t) = -\vec{k}$

(d) $\lim_{t \to \infty} \vec{r}(t) = -\vec{i} + 2\vec{j} - 2\vec{k}$

7.4

(a) $\vec{r}'(0) = 2\vec{i} - 2\vec{j} - 4\vec{k}$

(b) $\vec{r}'(0) = -2\vec{i} - 2\vec{k}$

7.5

(a) $-2t^2e^{-t} + 8te^{-t} - 4e^{-t} + 2e^{2t}(\cos 2t + \sin 2t)$

(b) $\left[2e^t - 2\{(t^2 - 2t)\cos 2t + (t-1)\sin 2t\}\right]\vec{k}$

7.6

(a) $t\vec{r}''(t) \times \vec{r}(t)$

(b) $e^t\,\vec{r}'(t) \times \vec{r}(t)$

(c) 0

(d) $2t\vec{r}''(t^2) + 2t^2\vec{r}'(t) + t\vec{r}(t^2)$

7.7

(a) $\vec{r}''(t) = (t^2e^t + 4te^t + 2e^t)\vec{i} + \dfrac{1}{t}\vec{j} + (-3e^{-t}\sin 2t - 4e^{-t}\cos 2t)\vec{k}$

$\vec{r}''(t)$를 한 번 더 미분하면 $\vec{r}'''(t)$가 구해진다.

(b) $\vec{r}''(t) = 4e^{-2t}\vec{i} - 4\sin 2t\,\vec{j} + 2\vec{k}$

$\vec{r}''(t)$을 한번 더 미분하면 $\vec{r}'''(t)$가 구해진다.

7.8

곡률 : $\kappa = \dfrac{1}{r}$

곡률반경 : $\rho = \dfrac{1}{\kappa} = r$

7.9

곡률 : $\kappa = \dfrac{1}{2}$

단위법선벡터 : $t = \dfrac{\pi}{2}$일 때 $\vec{N} = -\cos\pi/2\,\vec{i} - \sin\pi/2\,\vec{j} = -\vec{j}$

7.10

(a) $\vec{r}(t)=(t^2\cos t)\vec{i}+(t-3)\vec{j}$

$\vec{r}'(t)=(-t^2\sin t+2t\cos t)\vec{i}+\vec{j}$

$\vec{r}''(t)=(-t^2\cos t-4t\sin t+2\cos t)\vec{i}$

(b) $\vec{r}(t)=(2t^2-2)\vec{i}+\left(\dfrac{2}{t-1}\right)\vec{j}$

$\vec{r}'(t)=4t\,\vec{i}-\dfrac{2}{(t-1)^2}\,\vec{j}$

$\vec{r}'(t)=4\,\vec{i}+\dfrac{4}{(t-1)^3}\,\vec{j}$

(c) $\vec{r}(t)=(t^2e^{-t})\vec{i}+(2t\sin t-e^t)\vec{j}+(e^{-2t}\cos t)\vec{k}$

$\vec{r}'(t)=(2te^{-t}-t^2e^{-t})\vec{i}+(2\sin t+2t\cos t-e^t)\vec{j}+(-2e^{-2t}\cos t-e^{-2t}\sin t)\vec{k}$

$\vec{r}''(t)=(2e^{-t}-4te^{-t}+t^2e^{-t})\vec{i}+(4\cos t-2t\sin t-e^t)\vec{j}+(3e^{-2t}\cos t+4e^{-2t}\sin t)\vec{k}$

7.11

(a) $z_x=\dfrac{\partial z}{\partial x}=2x\cos y+15x^2y^2$

$z_y=\dfrac{\partial z}{\partial y}=-x^2\sin y+10x^3y+6y^2$

(b) $z_x=6x^2y+2xy^2+2y$

$z_y=2x^3+2x^2y+2x-3y^2$

(c) $z_x=\dfrac{2xy+2}{y^2+1}$

$z_y=\dfrac{2xy^2+2x-2x^2y^2-4xy}{(y^2+1)^2}$

(d) $z_x=\dfrac{\sqrt{\dfrac{y}{x}}(x^2+y^2)-2x(2\sqrt{xy}+y)}{(x^2+y^2)^2}$

$z_y=\dfrac{\left(\sqrt{\dfrac{x}{y}}+1\right)(x^2+y^2)-2y(2\sqrt{xy}+y)}{(x^2+y^2)^2}$

(e) $z_x=9\cos^2 2x(-2\sin 2x)-2x\sin^3 2y$

$z_y=-3x^2\sin^2 2y(2\cos 2x)$

(f) $z_x=2(x^2-2xy+2y^3)(2x-2y)$

$z_y=2(x^2-2xy+2y^3)(-2x+6y^2)$

7.12

(a) $z_{xx} = 2\cos y + 30xy^2$

$z_{yy} = -x^2\cos y + 10x^3 + 12y$

$z_{xy} = -2x\sin y + 30x^2 y$

(b) $z_{xx} = -12x^2y^2$

$z_{yy} = -2x^4 - 6xy$

$z_{xy} = 1 - 8x^3y - 3y^2$

(c) $z_{xx} = 8y^2\cos 4x$

$z_{yy} = 16x\cos 4y - 2\cos^2 2x$

$z_{xy} = 4\sin 4y(y+1)$

(d) $z_{xx} = 4(x^2 - xy + 2y^2) + 2(2x-y)^2 + 2$

$z_{yy} = 8(x^2 - xy + 2y^2) + 2(4y-x)^2 + 2$

$z_{xy} = -2(x^2 - xy + 2y^2) + 2(4y-x)(2x-y)$

7.13

(a) $\dfrac{\partial z}{\partial u} = (4x - 15x^2y^2)(\cos v + 2u\,e^{-v}) + (-10x^3y)(2v^2)$

$\dfrac{\partial z}{\partial v} = (4x - 15x^2y^2)(-u\sin v - u^2e^{-v}) + (-10x^3y)(4uv)$

(b) $\dfrac{\partial z}{\partial u} = (4xy - y^2)(-2u + 4uv) + (2x^2 - 2xy)(6u^2v + 2uv^2 + 2v)$

$\dfrac{\partial z}{\partial v} = (4xy - y^2)(2u^2 - 3v^2) + (2x^2 - 2xy)(2u^3 + 2u^2v + 2u)$

7.14

(a) $\dfrac{dz}{dt} = (-2ye^{-xy} - 2xy^2)(-2\sin 2t - 4e^{-2t}) + (-2xe^{-xy} - 2x^2y)(4t - \cos t)$

(b) $\dfrac{dz}{dt} = (-ye^{-x} - e^{-y})(e^{-2t}\sin t - 3e^{-2t}\cos t) + (e^{-x} + xe^{-y})(4t-1)$

7.15

(a) $\nabla f(1,2) = -4\vec{i} - 8\vec{j}$

(b) $\nabla f\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) = \pi\vec{j}$

(c) $\nabla f(2,\,1,\,1) = 3\vec{i} - 2\vec{j} - 2\vec{k}$

7.16

(a) $D_{\vec{a}} f = \dfrac{1}{\sqrt{2}}$

(b) $D_{\vec{a}} f = \dfrac{-24}{\sqrt{5}} + \dfrac{-12}{\sqrt{5}} = \dfrac{-36}{\sqrt{5}}$

(c) $D_{\vec{a}} f = \dfrac{6-2}{3} = \dfrac{4}{3}$

7.17

(a) 방향도함수의 최댓값 : $\| \nabla f \| = \sqrt{61}$

방향도함수의 최솟값 : $-\| \nabla f \| = -\sqrt{61}$

(b) 방향도함수의 최댓값 : $\| \nabla f \| = 12\sqrt{2}$

방향도함수의 최솟값 : $-\| \nabla f \| = -12\sqrt{2}$

(c) 방향도함수의 최댓값 : $\| \nabla f \| = \sqrt{37}$

방향도함수의 최솟값 : $-\| \nabla f \| = -\sqrt{37}$

7.18

$-20\vec{i} - 6\vec{j}$

7.19

$2\sqrt{41}$

7.20

(a) 3

(b) $2(x-y) - xz + ye^{2x}$

(c) $4xy + z$

(d) $e^{y} + xy$

(e) $y^2 z^2 + z$

(f) $e^{-z} + e^{-x} + e^{-y}$

(g) $2x\cos yz + 2y\sin xz + 2z$

7.21

$\operatorname{div} \vec{F} = 0$이므로 비압축성 유동

7.22

(a) $[xz-(xy+2z)]\vec{i}+(2y-yz)\vec{j}+(yz-2z)\vec{k}$

(b) $(x-x^2)\vec{i}+(y^2-y)\vec{j}+(2xz-2yz)\vec{k}$

(c) $-y\vec{i}-6x^2\vec{j}+(2x-2x^2)\vec{k}$

(d) $(xz-e^{-x})\vec{i}-yz\vec{j}+(e^{-x}z+xe^y)\vec{k}$

(e) $(2x^2y+x-y)\vec{i}+(2xy^2z-2xy^2)\vec{j}-(z+2xyz^2)\vec{k}$

(f) $-ze^{-y}\vec{i}-xe^{-z}\vec{j}-ye^{-x}\vec{k}$

(g) $-xy^2\cos xz\,\vec{i}-x^2y\sin yz\,\vec{j}+(y^2z\cos xz+x^2z\sin yz)\vec{k}$

7.23

$curl\ v=\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right)\vec{k}=\vec{0},$ 비회전적

7.24

증명 생략

Charter 8.

8.1

(a) $\displaystyle\int_C (xy-x)dx = \frac{16}{3}$

$\displaystyle\int_C (xy-x)dy = \frac{32}{3}$

$\displaystyle\int_C (xy-x)ds = \frac{16\sqrt{5}}{3}$

(b) $\displaystyle\int_C (x^2+xy+y^2)dx = 4$

$\displaystyle\int_C (x^2+xy+y^2)dy = -2$

$\displaystyle\int_C (x^2+xy+y^2)ds = 2\sqrt{5}$

(c) 0

8.2

(a) $-\dfrac{1120}{3}$

(b) $\dfrac{8}{5}$

(c) $\dfrac{88}{21}$

(d) $\dfrac{23}{30}$

8.3

(a) $-\dfrac{44}{3}$

(b) 22

(c) $\dfrac{16}{3}$

(d) 0

8.4

(a) $\dfrac{5}{12}$

(b) $-\dfrac{5}{12}$

8.5

$$\oint_C xy\,dx + x^2 dy = \frac{1}{2}$$

8.6

(a) $\displaystyle\int_C x\ dx + xy\ dy = -\frac{1}{6}$

(b) $\displaystyle\int_{-C} x\ dx + xy\ dy = \frac{1}{6}$

8.7

$$\int_C z\,dx + x\,dy + y\,dz = -\pi$$

8.8

(a) $W = 0$

(b) $W = -4$

8.9

$$W = \frac{27}{2}$$

8.10

32

8.11

8π

8.12

0

8.13

$$\frac{133}{3}$$

8.14

(a) $\int_{C_1} xy\,dx - y^2\,dy = -2$

(b) $\int_{C_2+C_3} xy\,dx - y^2\,dy = -\frac{8}{3}$

(a)와 (b)의 결과를 보면 동일한 함수의 선적분 값이 경로에 따라 다르게 나타남을 알 수 있다.

8.15

(a) 2

(b) 2

(c) 2

8.16

(a) $\int_C xy^2\,dx + x^2y\,dy = 8$

(b) $\int_C -2xy\,dx - x^2\,dy = -8$

8.17

(a) $\int_C 2xz\,dx + 2yz\,dy + (x^2+y^2)dz = 4$

(b) $\int_C yz\,dx + xz\,dy + xydz = 2$

8.18

(a) $\phi = \frac{1}{2}x^2 + yx - \frac{1}{2}y^2 + C$

(b) $\phi(x,y) = \frac{1}{3}x^2 + x^2y^2 + x - y^2 - 3y + C$

(c) $\phi = x^2\cos y + \frac{1}{2}x^2 - yx - y + C$

(d) $\phi = xyz$

(e) $\phi = x^2y + xz + \frac{1}{2}z^2 + yz - \frac{1}{2}z^2 + c = x^2y + xz + yz + C$

8.19

$2(e+e^{-1})$

8.20

$\frac{3}{20}$

8.21

$\frac{3}{20}$

8.22

$\frac{\pi}{16}$

8.23

$\frac{11}{30}$

8.24

10π

8.25

$\iint_S f(x, y, z)\, dS \approx 563.7$

8.26

증명 생략

8.27

$\frac{13}{12}$

8.28

증명 생략

Charter 9.

9.1

(a) $a = -3,\ b = 4$

(b) $a = 2,\ b = 1$

9.2

(a) $2\overrightarrow{A} - 4\overrightarrow{B} = \begin{bmatrix} 4 & -18 & -8 \\ 0 & -6 & 12 \\ -6 & 12 & -14 \end{bmatrix}$

(b) $-2\overrightarrow{A} - \overrightarrow{B} = \begin{bmatrix} -2 & -2 & -2 \\ 5 & -9 & -2 \\ -4 & -2 & -1 \end{bmatrix}$

(c) $2(\overrightarrow{A} + 2\overrightarrow{B}) = \begin{bmatrix} 4 & 14 & 8 \\ -8 & 18 & -4 \\ 10 & -4 & 10 \end{bmatrix}$

(d) $2\overrightarrow{A} + \overrightarrow{B} - \overrightarrow{C} = \begin{bmatrix} 1 & 3 & 0 \\ -4 & 7 & 4 \\ 3 & 5 & 1 \end{bmatrix}$

(e) $-2\overrightarrow{A} - (\overrightarrow{B} + 2\overrightarrow{C}) = \begin{bmatrix} -10 & 0 & -6 \\ 7 & -13 & 2 \\ -6 & 4 & -1 \end{bmatrix}$

(f) $-2(\overrightarrow{A} - \overrightarrow{B}) - \overrightarrow{C} = \begin{bmatrix} -7 & 11 & 2 \\ 3 & -2 & -6 \\ 1 & 5 & 8 \end{bmatrix}$

(g) $-\overrightarrow{A} + \overrightarrow{B^T} + \overrightarrow{C} = \begin{bmatrix} 4 & -1 & 4 \\ 7 & 2 & -6 \\ 2 & -7 & 4 \end{bmatrix}$

(h) $(2\overrightarrow{A} - \overrightarrow{B})^T - \overrightarrow{C} = \begin{bmatrix} 1 & -2 & -2 \\ -5 & 1 & 8 \\ -3 & 9 & -5 \end{bmatrix}$

(i) $(\overrightarrow{A} + \overrightarrow{B} + \overrightarrow{C})^T = \begin{bmatrix} 5 & -4 & 4 \\ 2 & 8 & -3 \\ 4 & -2 & 2 \end{bmatrix}$

9.3

(a) $\overrightarrow{A}\,\overrightarrow{B} = \begin{bmatrix} 1 & 11 \\ -3 & -17 \end{bmatrix}$

(b) $\overrightarrow{B}\,\overrightarrow{A} = \begin{bmatrix} -8 & 12 \\ 4 & -8 \end{bmatrix}$

(c) $\overrightarrow{C}(\overrightarrow{A}\,\overrightarrow{B}) = \begin{bmatrix} 6 & 50 \\ -7 & -45 \end{bmatrix}$

(d) $(\overrightarrow{A}\,\overrightarrow{B})\overrightarrow{C} = \begin{bmatrix} -8 & 21 \\ 8 & -31 \end{bmatrix}$

(e) $\overrightarrow{A}\,(\overrightarrow{B}\,\overrightarrow{C}) = \begin{bmatrix} -8 & 21 \\ 8 & -31 \end{bmatrix}$

(f) $(\overrightarrow{C}\,\overrightarrow{A})\overrightarrow{B} = \begin{bmatrix} 6 & 50 \\ -7 & -45 \end{bmatrix}$

(g) $\overrightarrow{A^T}\overrightarrow{B}=\begin{bmatrix} 2 & 14 \\ -3 & -13 \end{bmatrix}$

(h) $\overrightarrow{A^T}\,\overrightarrow{B^T}\,\overrightarrow{C^T}=\begin{bmatrix} -28 & 16 \\ 44 & -28 \end{bmatrix}$

(i) $(\overrightarrow{A}\,\overrightarrow{B}\,\overrightarrow{C})^T=\begin{bmatrix} -8 & 8 \\ 21 & -31 \end{bmatrix}$

9.4

(a) 3

(b) 2

(c) 1

9.5

(a) -1

(b) 5

(c) 1

9.6

풀이 생략

9.7

풀이 생략

9.8

풀이 생략

9.9

풀이 생략

9.10

풀이 생략

9.11

$\overrightarrow{A^{-1}}=\dfrac{1}{13}\begin{bmatrix} -4 & -5 \\ 1 & -2 \end{bmatrix}$, $\quad\overrightarrow{B^{-1}}=\dfrac{1}{11}\begin{bmatrix} 9 & 7 & 6 \\ 7 & 3 & 1 \\ -6 & -1 & -4 \end{bmatrix}$

9.12

$\det\overrightarrow{A} = 0$, $\therefore$ 행렬 $\overrightarrow{A}$는 특이행렬

Charter 10.

10.1

(a) $\begin{bmatrix} -1 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 3 \end{bmatrix}$

10.2

$-x+y-2z=1$

$-2y+4z=2$

$-x-y+2z=3$

10.3

(a) $x_1 = 2.2$, $x_2 = 1.3$

(b) $x_1 = \frac{11}{8}$, $x_2 = \frac{1}{16}$

(c) 해가 존재하지 않는다.

(d) $x_1 = \frac{7}{15}$, $x_2 = -\frac{2}{15}$, $x_3 = -\frac{4}{5}$

(e) $x_1 = 1$, $x_2 = 4$, $x_3 = 10$

(f) 해가 존재하지 않는다.

10.4

(a) $x_1 = -2$, $x_2 = -\frac{5}{2}$

(b) $x_1 = -\frac{1}{6}$, $x_2 = \frac{5}{3}$

(c) $x_1 = -\frac{14}{3}$, $x_2 = -\frac{5}{3}$

(d) $x_1 = \frac{18}{5}$, $x_2 = 2$, $x_3 = -\frac{4}{5}$

(e) $x_1 = -\frac{2}{3}$, $x_2 = \frac{3}{4}$, $x_3 = \frac{4}{3}$

(f) $x_1 = -\frac{14}{3}$, $x_2 = -\frac{4}{3}$, $x_3 = 5$

10.5

(a) $x_1 = -\frac{1}{11}, \quad x_2 = -\frac{3}{11}$

(b) $x_1 = \frac{1}{4}, \quad x_2 = -\frac{1}{2}$

(c) 해가 존재하지 않는다.

(d) $x_1 = -\frac{14}{3}, \quad x_2 = -\frac{4}{3}, \quad x_3 = 5$

(e) $x_1 = \frac{5}{2},\ x_2 = -\frac{5}{6},\ x_3 = -\frac{1}{6}$

(f) $x_1 = -\frac{7}{5} - \frac{4}{5}c,\ x_2 = -\frac{4}{5} - \frac{3}{5}c,\ x_3 = c$

(g) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1+c \\ c \\ 0 \end{pmatrix}$ c는 임의의 상수

(h) 해가 존재하지 않는다.

(i) 해가 존재하지 않는다.

10.6

(a) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} c$

(b) $x_1 = x_2 = 0$ (자명한 해)

(c) $x_1 = x_2 = 0$ (자명한 해)

(d) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 3 \end{bmatrix} c$

(e) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{6}{7} \\ \frac{4}{7} \\ 1 \end{pmatrix} c$

(f) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix} c$

(g) i) $x_3 = 0$이면, $x_1 = x_2$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} c$

ii) $x_2 = 0$이면, $x_1 = -x_3$, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} c$

(h) $x_2 = c_2$, $x_3 = c_3$로 가정하면, 연립방정식의 해 : $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}c_2 + c_3 \\ c_2 \\ c_3 \end{pmatrix}$

또는 $x_2 = c$, $x_3 = 0$으로 가정하면 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}c \\ c \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \\ 0 \end{pmatrix}c$ ...... ①

$x_2 = 0$, $x_3 = c$로 가정하면 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} c \\ 0 \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}c$ ...... ②

식 ①과 식 ②는 서로 일차독립이므로 둘 다 주어진 선형 연립방정식의 해가 될 수 있다.

(i) $x_2 = -2c$, $x_3 = 0$으로 가정하면 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}c$ ...... ①

$x_2 = 0$, $x_3 = 2c$로 가정하면 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}c$ ...... ②

식 ①과 식 ②는 서로 일차독립이므로 둘 다 주어진 선형 연립방정식의 해가 될 수 있다.

(j) $x_1 = x_2 = x_3 = 0$(자명한 해)

(k) $x_1 = x_2 = x_3 = 0$(자명한 해)

(l) $x_3 = c$(0이 아닌 상수)라 가정하면 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{13}{4} \\ 1 \end{pmatrix}c$

10.7

(a) $\overrightarrow{A}^{-1} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

(b) $\overrightarrow{A}^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{14} & -\frac{3}{14} & \frac{1}{7} \\ \frac{1}{7} & \frac{4}{7} & \frac{2}{7} \end{bmatrix}$

(c) $\overrightarrow{A}^{-1} = \begin{bmatrix} \frac{7}{2} & 5 & -\frac{1}{2} \\ -\frac{3}{2} & -2 & \frac{1}{2} \\ -\frac{5}{2} & -4 & \frac{1}{2} \end{bmatrix}$

(d) $\overrightarrow{A}^{-1} = \begin{bmatrix} \frac{3}{5} & 1 & \frac{1}{5} & \frac{2}{5} \\ -1 & -1 & 1 & -1 \\ -\frac{2}{5} & -1 & \frac{1}{5} & -\frac{3}{5} \\ \frac{4}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

10.8

(a) $i_1 = \frac{10}{2R_1 + R_2}$, $i_2 = \frac{5}{2R_1 + R_2}$, $i_3 = \frac{5}{2R_1 + R_2}$

(b) $i_1 = 16/11\,A$, $i_2 = 10/11\,A$, $i_3 = 6/11A$

10.9

(a)

1) $\lambda_1 = -1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = 1$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} c$$

(b)

1) $\lambda_1 = 3$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = -1$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix} c$$

(c)

1) $\lambda_1 = 0$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = -5$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} c$$

(d)

$\lambda_1 = \lambda_2 = 4$인경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} c$$

(e)

$\lambda_1 = \lambda_2 = 2$인 경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{1,\ 2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} c$$

(f)

$\lambda_1 = \lambda_2 = -1$인경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} c$$

(g)

1) $\lambda_1 = 1 + i$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} i \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = \overline{\lambda_1} = 1 - i$인 경우

$$\overrightarrow{V_2} = \overline{\overrightarrow{V_1}} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix} c$$

(h)

1) $\lambda_1 = 4 + i$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -2 - i \end{bmatrix} c$$

2) $\lambda_2 = 4 - i$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_i = \begin{bmatrix} 1 \\ -2 + i \end{bmatrix} c$$

(i)

1) $\lambda_1 = 2i$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ -1 + i \end{bmatrix} c$$

2) $\lambda_2 = -2i$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_i = \begin{bmatrix} 1 \\ -1 - i \end{bmatrix} c$$

(j)

1) $\lambda_1 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$$

2) $\lambda_2 = \sqrt{2}$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} \sqrt{2} \\ 1 \\ \sqrt{2}-1 \end{bmatrix} c$$

3) $\lambda_3 = -\sqrt{2}$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} -\sqrt{2} \\ 1 \\ -\sqrt{2}-1 \end{bmatrix} c$$

(k)

1) $\lambda_1 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = -1$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} c$$

3) $\lambda_3 = 3$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} c$$

(l)

1) $\lambda_1 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = 2$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} c$$

3) $\lambda_3 = 3$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} c$$

(m)

1) $\lambda_1 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = \lambda_3 = 2$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} , \ \overrightarrow{V_3} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

(n)

1) $\lambda_1 = 7$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = \lambda_3 = 4$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} c \ , \ \overrightarrow{V_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} c$$

(o)

1) $\lambda_1 = \lambda_2 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} c, \quad \overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} c$$

2) $\lambda_3 = 4$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} c$$

(p)

1) $\lambda_1 = \lambda_2 = \lambda_3 = 1$인 경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} c$$

(q)

1) $\lambda_1 = \lambda_2 = \lambda_3 = -2$인 경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} c$$

(r)

1) $\lambda_1 = \lambda_2 = \lambda_3 = 2$인 경우

$$\overrightarrow{V_1} = \overrightarrow{V_2} = \overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} c$$

(s)

1) $\lambda_1 = -3$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = 1+2i$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} -3i \\ -2-i \\ 2 \end{bmatrix} c$$

3) $\lambda_3 = 1-2i$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 3i \\ -2+i \\ 2 \end{bmatrix} c$$

(t)

1) $\lambda_1 = 1$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} c$$

2) $\lambda_2 = 1+i$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ -i \\ 1 \end{bmatrix} c$$

3) $\lambda_3 = 1-i$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 1 \\ i \\ 1 \end{bmatrix} c$$

(u)

1) $\lambda_1 = -2$인 경우

$$\overrightarrow{V_1} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} c$$

2) $\lambda_2 = 2i$인 경우

$$\overrightarrow{V_2} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_2 = \begin{bmatrix} 1 \\ 1 \\ -2-2i \end{bmatrix} c$$

3) $\lambda_3 = -2i$인 경우

$$\overrightarrow{V_3} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}_3 = \begin{bmatrix} 1 \\ 1 \\ -2+2i \end{bmatrix} c$$

10.10

(a) $\overrightarrow{D} = \overrightarrow{P}^{-1} \overrightarrow{A}\,\overrightarrow{P} = \dfrac{1}{4}\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

(b) $\overrightarrow{D} = \overrightarrow{P}^{-1} \overrightarrow{A}\,\overrightarrow{P} = \dfrac{1}{2}\begin{bmatrix} -i & 1 \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+i & 0 \\ 0 & 1-i \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

(c) 대각화가 불가능하다.

(d) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(e) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 1+i & 0 & 0 \\ 0 & 1-i & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(f) $\overrightarrow{D} = \overrightarrow{P}^{-1} \overrightarrow{A}\,\overrightarrow{P} = \dfrac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

(g) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(h) 대각화가 불가능하다.

(i) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 1+2i & 0 & 0 \\ 0 & 1-2i & 0 \\ 0 & 0 & -3 \end{bmatrix}$

10.11

(a) $\overrightarrow{D} = \overrightarrow{P}^{-1} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

(b) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $\overrightarrow{D} = \overrightarrow{P^{-1}} \overrightarrow{A}\,\overrightarrow{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

10.12

(a) 고유벡터가 모두 서로 직교한다.

(b) $\overrightarrow{V}_1$과 $\overrightarrow{V}_2$는 직교하지 않는다.

$\overrightarrow{V}_2$와 $\overrightarrow{V}_3$는 직교한다.

$\overrightarrow{V}_1$과 $\overrightarrow{V}_3$도 직교한다.

10.13

(a) $\overrightarrow{A}^4 = \begin{bmatrix} 417 & 208 \\ 416 & 209 \end{bmatrix}$

(b) $\overrightarrow{A}^4 = \begin{bmatrix} 41 & 60 & 80 \\ 0 & 1 & 0 \\ 20 & 30 & 41 \end{bmatrix}$

(c) $\overrightarrow{A}^4 = \begin{bmatrix} 4 & -1 & -3 \\ 5 & 6 & -5 \\ -7 & -11 & 8 \end{bmatrix}$

10.14

(a) $\overrightarrow{A}^{-1} = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

(b) $\overrightarrow{A}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

Charter 11.

11.1

(a) $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(b) $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(c) $\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(d) $\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 2 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(e) $\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ -1 & -1 & -1 \\ -2 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(f) $\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ -3 & -2 & -1 \\ 2 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

11.2

(a) $\frac{dx}{dt} = -x + 2y, \ \frac{dy}{dt} = -2x - y$

(b) $\frac{dx}{dt} = x - 2y, \quad \frac{dy}{dt} = 3x - 2y$

(c) $\frac{dx}{dt} = -x + 2y + z, \ \frac{dy}{dt} = x - 2y + 2z, \ \frac{dz}{dt} = 3y - z$

(d) $\frac{dx}{dt} = -x + 2y, \ \frac{dy}{dt} = 2x - y - 2z, \ \frac{dz}{dt} = -x + 3y + 3z$

11.3

증명 생략

11.4

(a) $x = -c_1 + c_2 e^{-3t}, \quad y = c_1 + 2c_2 e^{-3t}$

(b) $x = c_1 e^{t} - c_2 e^{-t}, \quad y = -3c_1 e^{t} + c_2 e^{-t}$

(c) $x = -2c_1 + c_2 e^{-5t}, \quad y = c_1 + 3c_2 e^{-5t}$

(d) $x = c_1 e^{4t} + c_2 t e^{4t}, \quad y = -\frac{1}{2} c_2 e^{4t}$

(e) $x = (c_1 + c_2 t) e^{2t}, \quad y = (c_1 + c_2 + c_2 t) e^{2t}$

(f) $x = C_1 e^{2t} \sin t + C_2 e^{2t} \cos t, \quad y = (C_1 + C_2) e^{2t} \sin t + (C_2 - C_1) e^{2t} \cos t$

(g) $\overrightarrow{X} = C_1 \overrightarrow{Y_1} + C_2 \overrightarrow{Y_2} = c_1 \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos 2t + \begin{bmatrix} 0 \\ 2 \end{bmatrix} \sin 2t \right\} e^{-t} + c_2 \left\{ \begin{bmatrix} 0 \\ -2 \end{bmatrix} \cos 2t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin 2t \right\} e^{-t}$

(h) $x = (-c_1 - c_2)\cos 2t + (c_1 - c_2)\sin 2t, \quad y = 2c_1\cos 2t + 2c_2\sin 2t$

(i) $x = -3c_1e^{-2t} + c_3e^{2t}, \quad y = 2c_1e^{-2t} - c_2e^{t} + 2c_3e^{2t}, \quad z = c_1e^{-2t} + c_2e^{t} + c_3e^{2t}$

(j) $x = -c_2e^{2t}, \quad y = c_1e^{t} + 3c_2e^{2t} + c_3e^{3t}, \quad z = c_1e^{t} + c_2e^{2t}$

(k) $x = c_1\sqrt{2}\,e^{\sqrt{2}t} - c_2\sqrt{2}\,e^{-\sqrt{2}t} + c_3e^{t}$

$y = c_1e^{\sqrt{2}t} + c_2e^{-\sqrt{2}t}$

$z = c_1(\sqrt{2}-1)e^{\sqrt{2}t} - c_2(\sqrt{2}+1)e^{-\sqrt{2}t}$

(l) $x = c_1e^{5t} - c_2e^{2t} - c_3e^{2t}$

$y = c_1e^{5t} + c_2e^{2t}$

$z = c_1e^{5t} + c_3e^{2t}$

(m) $x = c_1e^{t} + c_2e^{2t} + c_3e^{2t}$

$y = c_1e^{t} + c_2e^{2t}$

$z = c_1e^{t} + c_3e^{2t}$

(n) $x = c_3\,e^{t}$

$y = c_2\,e^{t} + c_3\,t\,e^{t} - 2c_3\,e^{t}$

$z = c_1\,e^{t} + c_2\,t\,e^{t} + c_3\,\dfrac{t^2}{2}\,e^{t}$

(o) $x = c_1e^{t} + c_2te^{t} + c_2e^{t} + c_3\dfrac{t^2}{2}e^{t} + c_3te^{t}$

$y = c_3\dfrac{1}{2}e^{t}$

$z = c_1e^{t} + c_2te^{t} + c_3\dfrac{t^2}{2}e^{t}$

(p) $x = -2c_1e^{-3t} - \dfrac{3}{2}c_2e^{t}\cos 2t + \dfrac{3}{2}c_3e^{t}\sin t$

$y = \dfrac{1}{2}e^{t}(-2c_2\sin 2t - c_2\cos 2t - 2c_3\cos 2t + c_3\sin 2t)$

$z = c_1e^{-3t} + c_2e^{t}\sin 2t + c_3\,e^{t}\cos 2t$

(q) $x = e^{t}(c_2\cos t - c_3\sin t + c_1)$

$y = e^{t}(c_2\sin t + c_3\cos t)$

$z = \dfrac{1}{2}e^{t}(2c_2\cos t - 2c_3\sin t + c_1)$

11.5

(a) $\overrightarrow{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overrightarrow{P}\,\overrightarrow{W} = \begin{bmatrix} 4 & -2 & 2 \\ -4 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} c_1e^{t} \\ c_2e^{-t} \\ c_e e^{3t} \end{bmatrix} = \begin{bmatrix} 4c_1e^{t} - 2c_2e^{-t} + 2c_3e^{3t} \\ -4c_1e^{t} \\ c_1e^{t} + c_2e^{-t} + c_3e^{3t} \end{bmatrix}$

(b) $\overrightarrow{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \overrightarrow{P}\,\overrightarrow{W} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} c_1e^{5t} \\ c_2e^{5t} \\ c_e e^{8t} \end{bmatrix} = \begin{bmatrix} -c_1e^{5t} - c_2e^{5t} + c_3e^{8t} \\ c_1e^{5t} + c_3e^{8t} \\ c_2e^{5t} + c_3e^{8t} \end{bmatrix}$

11.6

(a) $\overrightarrow{X} = \begin{bmatrix} x \\ y \end{bmatrix} = c_1\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} 1 \\ 2 \end{bmatrix}e^{6t} + \begin{bmatrix} -\dfrac{13}{18} \\ \dfrac{8}{9} \end{bmatrix}$

(b) $\overrightarrow{X} = \overrightarrow{X}_c + \overrightarrow{X}_p = c_1\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{-2t} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{4t} + \begin{bmatrix} -\dfrac{9}{8} \\ \dfrac{11}{8} \end{bmatrix}$

(c) $\overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P} = c_1\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{-t} + c_2\begin{bmatrix} 1 \\ -3 \end{bmatrix}e^{t} + \begin{bmatrix} 2t-3 \\ -3t+4 \end{bmatrix}$

(d) $\overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P} = c_1\begin{bmatrix} 1 \\ -3 \end{bmatrix}e^{-5t} + c_2\begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} -\dfrac{1}{26}\cos t + \dfrac{31}{26}\sin t + \dfrac{4}{5}t + \dfrac{4}{5} \\ \dfrac{3}{26}\cos t - \dfrac{15}{26}\sin t - \dfrac{2}{5}t \end{bmatrix}$

(e) $\overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P} = c_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{5t} + c_2\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}te^{5t} + \begin{bmatrix} 0 \\ -\dfrac{1}{2} \end{bmatrix}e^{5t}\right\} + \begin{bmatrix} -\dfrac{3}{5} + \dfrac{1}{18}e^{-t} \\ \dfrac{1}{6}e^{-t} \end{bmatrix}$

(f) $\overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P} = c_1\begin{bmatrix} \cos t \\ \sin t - 2\cos t \end{bmatrix}e^{4t} + c_2\begin{bmatrix} \sin t \\ -2\sin t - \cos t \end{bmatrix}e^{4t} + \begin{bmatrix} -\dfrac{16}{17} \\ \dfrac{19}{17} \end{bmatrix}$

11.7

이 문제에서 $\overrightarrow{X_c}$ 구하는 방법은 11.6문제와 동일하므로 생략하고 $\overrightarrow{X_P}$만 구하고자 한다.

(a) $\overrightarrow{X_P} = \begin{bmatrix} -\dfrac{13}{18} \\ \dfrac{8}{9} \end{bmatrix}$

[연습문제 11.6] (a)의 결과와 동일하다.

(b) $\overrightarrow{X_P} = \begin{bmatrix} -\dfrac{9}{8} \\ \dfrac{11}{8} \end{bmatrix}$

(c) ~ (f)는 생략한다.

11.8

(a) $\overrightarrow{X_c} = c_1\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}e^{-t} + c_3\begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}e^{t}$

$\overrightarrow{X_P} = \begin{bmatrix} \dfrac{23}{9} - \dfrac{4}{3}t \\ 0 \\ -\dfrac{11}{9} + \dfrac{1}{3}t \end{bmatrix}, \quad \therefore \overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P}$

(b) $\overrightarrow{X_c} = c_1\begin{bmatrix}-6\\1\\1\end{bmatrix}e^t + c_2\begin{bmatrix}1\\ \frac{3}{2}\\ -\frac{5}{2}\end{bmatrix}e^{5t} + c_3\begin{bmatrix}0\\1\\1\end{bmatrix}e^{7t}$

$$\overrightarrow{X_P} = \begin{bmatrix}\frac{2}{5}+\frac{1}{4}e^{3t}\\ -\frac{2}{5}-\frac{3}{16}e^{3t}\\ \frac{5}{16}e^{3t}\end{bmatrix}, \quad \therefore \overrightarrow{X} = \overrightarrow{X_c} + \overrightarrow{X_P}$$

11.9

(a) $x = -c_1 e^{-t} + c_2 e^{4t} - \frac{5}{6}e^t, \quad y = c_1 e^{-t} + \frac{3}{2}c_2 e^{4t} + \frac{1}{2}e^t$

(b) $x = -c_1 e^{-2t} + c_2 e^{4t} - \frac{9}{8}, \quad y = c_1 e^{-2t} + c_2 e^{4t} + \frac{11}{8}$

(c) $x = c_1 e^{3t} + c_2 e^{-t} - \frac{2}{3} + \frac{2}{5}e^{4t}$

$y = c_1 e^{3t} - c_2 e^{-t} + \frac{4}{3} + \frac{3}{5}e^{4t}$

$z = c_3 e^{2t} - \frac{1}{4} - \frac{1}{2}t$

(d) $x = -c_1 e^{-2t} - 2c_2 e^{-2t} + c_3 e^t + \frac{3}{10}e^t + 2$

$y = c_2 e^{-2t} + c_3 e^t + \frac{1}{10}e^{3t}$

$z = c_1 e^{-2t} + c_2 e^{-2t} + c_3 e^t + \frac{1}{10}e^{3t} + 1$

11.10

(a) $i_1 = 10t + c_1, \quad i_3 = c_2 e^{-t}, \quad i_2 = i_1 - i_3$에 의해 구함

(b) $i_1 = -\frac{1}{2}c_1 e^{-2t} + c_2 + 5t$

$i_2 = \frac{1}{2}c_1 e^{-2t} + c_2 + 5t - 5$

$i_3 = i_1 - i_2$

Charter 12.

12.1
풀이 생략

12.2
$f_0(x)=1, f_1(x)=\sin x, f_2(x)=\sin 2x$는 구간 $\left[0, \frac{\pi}{2}\right]$에서 서로 직교한다.

$$\therefore \quad \frac{f_0(x)}{\sqrt{\frac{\pi}{2}}}=\frac{1}{\sqrt{\frac{\pi}{2}}}=\frac{\sqrt{2}}{\sqrt{\pi}}, \quad \frac{f_1(x)}{\frac{\sqrt{\pi}}{2}}=\frac{2\sin x}{\sqrt{\pi}}, \quad \frac{f_2(x)}{\frac{\sqrt{\pi}}{2}}=\frac{2\sin 2x}{\sqrt{\pi}}$$ 는 서로 정규직교함수이다.

12.3
$$f(x)=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1-\cos n\pi}{n\pi}\sin nx$$

12.4
$$f(x)=\frac{\pi}{4}+\sum_{n=1}^{\infty}\left(\frac{\cos n\pi-1}{\pi n^2}\cos nx-\frac{\cos n\pi}{n}\sin nx\right)$$

12.5
$$f(x)=\sum_{n=1}^{\infty}\frac{-2}{n\pi}\cos n\pi\,\sin\frac{n\pi}{2}x$$

12.6
$$f(x)=\frac{1}{\pi}+\sum_{n=1}^{\infty}\left[\frac{2(-1)^{n+1}}{\pi(4n^2-1)}\cos 2nx+\frac{4n}{\pi(4n^2-1)}\sin 2nx\right]$$

12.7
$$f(x)=\frac{\pi}{4}+\sum_{n=1}^{\infty}\left[\frac{3(1-\cos n\pi)}{\pi n^2}\cos nx+\frac{1}{n}\cos n\pi sinnx\right]$$

12.8
(a) $f(x)=\sum_{n=1}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi}{2}\cos\frac{n\pi}{2}x$

(b) $f(x)=\sum_{n=1}^{\infty}\left(-\frac{4}{n\pi}\sin\frac{n\pi}{2}\cos\frac{n\pi}{2}x\right)$

(c) $f(x) = \dfrac{\pi}{2} + \sum_{n=1}^{\infty} \dfrac{2}{\pi n^2}(1 - \cos n\pi)\cos nx$

(d) $f(x) = \dfrac{\pi}{2} + \sum_{n=1}^{\infty} \dfrac{2}{\pi n^2}\left[(-1)^n - 1\right]\cos nx$

12.9

(a) $f(x) = \sum_{n=1}^{\infty}\left[-\dfrac{2}{n}(-1)^n\right]\sin nx = \sum_{n=1}^{\infty}\dfrac{2}{n}(-1)^{n+1}\sin nx$

(b) $f(x) = \sum_{n=1}^{\infty}\left[\dfrac{2}{n}(-1)^n\right]\sin nx$

(c) $f(x) = \sum_{n=1}^{\infty}\left[\dfrac{2\pi^2}{n}(-1)^{n+1} + \dfrac{12}{n^3}(-1)^n\right]\sin nx$

12.10

(a) $f(x) = \dfrac{1}{2} - \sum_{n=1}^{\infty}\dfrac{2}{n\pi}\sin\dfrac{n\pi}{2}\cos\dfrac{n\pi}{2}x$

(b) $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\dfrac{n\pi}{T}x = \dfrac{1}{2} + \sum_{n=1}^{\infty}\dfrac{2}{(n\pi)^2}(\cos n\pi - 1)\cos n\pi x$

12.11

(a) $f(x) = \sum_{n=1}^{\infty}\dfrac{2}{n\pi}\left(\cos\dfrac{n\pi}{2} - \cos n\pi\right)\sin\dfrac{n\pi}{2}x$

(b) $f(x) = \sum_{n=1}^{\infty} b_n \sin\dfrac{n\pi}{T}x = \sum_{n=1}^{\infty}\dfrac{-2}{n\pi}\cos n\pi\,\sin n\pi x$

12.12

(a) $f(x) = \sum_{n=-\infty}^{\infty}\dfrac{i}{2\pi n}(\cos n\pi - 1)e^{inx}$

(b) $f(x) = \sinh(1)\sum_{n=-\infty}^{\infty}\dfrac{1 + in\pi}{1 + (n\pi)^2}(-1)^n e^{in\pi x}$

12.13

$f(x) = \dfrac{1}{\pi}\int_0^{\infty}\left[\dfrac{\sin\omega x + \sin(\omega - \omega x)}{\omega}\right]d\omega$

12.14

(a) $f(x) = \dfrac{2}{\pi}\displaystyle\int_0^{\infty} \dfrac{2}{2^2 + w^2} \cos \omega x \, d\omega$

(b) $A(w) = \dfrac{1}{\pi}\left[\dfrac{-1}{1+w} \cos\pi(1+w) - \dfrac{1}{1-w} \cos\pi(1-w) + \dfrac{1}{1+w} + \dfrac{1}{1-w} \right]$

$f(x) = \displaystyle\int_0^{\infty} A(w) \cos wx \, dw$

12.15

(a) $f(x) = \dfrac{2}{\pi}\displaystyle\int_0^{\infty} \dfrac{\omega}{2^2 + w^2} \sin \omega x \, d\omega$

(b) $B(w) = \dfrac{1}{\pi}\left[\dfrac{1}{1-w} \sin\pi(1-w) - \dfrac{1}{1+w} \sin\pi(1+w) \right]$

$f(x) = \displaystyle\int_0^{\infty} B(w) \sin wx \, dw$

12.16

(a) $\hat{f}(w) = \dfrac{c}{iw}\left(1 - e^{-2wi}\right)$

(b) $\hat{f}(w) = -\dfrac{c}{iw} e^{-iwa} + \dfrac{c}{iw} e^{-iwa} = \dfrac{c}{iw}\left(e^{iwa} - e^{-iwa}\right) = 2c\,\dfrac{\sin a\omega}{\omega}$

(c) $\hat{f}(w) = \dfrac{1}{2 + i\omega}$

(d) $\hat{f}(w) = \dfrac{1}{1 + \omega^2}$

12.17

$\hat{g}(\omega) = \mathscr{F}\{f(t-4)\} = e^{-4i\omega} \hat{f}(\omega) = e^{-4i\omega} \dfrac{2c}{\omega} \sin(2\omega)$

Charter 13.

13.1

(a) 포물형

(b) 쌍곡형

(c) 타원형

13.2

증명 생략

13.3

(a) $u(x,\ y) = c_1(y)\cos x + c_2(y)\sin x$

(b) $u(x,\ y) = e^y \int f(x)\,dx + g(y)$

(c) $u = F(x)\,G(y) = c_1 e^{2\alpha x} c_2 e^{\alpha y} = Ae^{\alpha(2x+y)}$

(d) $u = F(x)\,G(y) = c_1 e^{\alpha x} c_2 e^{\frac{1}{2}(1-\alpha)y} = Ae^{\alpha x} e^{\frac{1}{2}(1-\alpha)y}$

13.4

(a) $u(x,\ t) = \sum_{n=1}^{\infty}\left(\frac{2L}{n\pi}\cos n\pi \cos\frac{cn\pi}{L}t\right)\sin\frac{n\pi}{L}x$

(b) $u(x,\ t) = \sum_{n=1}^{\infty}\left[2\left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi}{2}\cos\frac{cn\pi}{L}t\right]\sin\frac{n\pi}{L}x$

(c) $u(x,\ t) = \sum_{n=1}^{\infty}\left(A_n\cos\frac{cn\pi}{\pi}t + B_n\sin\frac{cn\pi}{\pi}t\right)\sin\frac{n\pi}{\pi}x = \frac{0.1}{c}\sin 2ct\,\sin 2x$

13.5

(a) $u(x,\ t) = \sum_{n=1}^{\infty}\frac{2}{n\pi}(1-\cos n\pi)\sin\frac{n\pi}{L}x\ e^{-\left(\frac{cn\pi}{L}\right)^2 t}$

(b) $u(x,\ t) = \sum_{n=1}^{\infty}\left(-\frac{L^2}{n\pi}\cos n\pi\right)\sin\frac{n\pi}{L}x\ e^{-\left(\frac{cn\pi}{L}\right)^2 t}$

(c) $u(x,\ t) = \sum_{n=1}^{\infty}\frac{L^2}{n\pi}\sin\frac{n\pi}{L}x\ e^{-\left(\frac{cn\pi}{L}\right)^2 t}$

13.6

(a) $u(x, y) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi}{a}x \cosh\frac{n\pi}{a}y$, 여기서 $A_n = \frac{2}{a\cosh\frac{n\pi b}{a}}\int_0^a f(x)\sin\frac{n\pi}{a}x\,dx$

(b) $-\frac{2a}{n\pi}\frac{\cos n\pi}{\cosh(n\pi b/a)}$

(c) $\frac{2a}{n\pi}\frac{1}{\cosh(n\pi b/a)}$

13.7

(a) $v(x, t) = u(t) + t - (t-x)u(t-x)$

(b) $v(x, t) = \frac{1}{2}t^2 - \frac{1}{2}(t-x)^2 u(t-x)$

Charter 14.

14.1

(a) $\sqrt{3}+i$

(b) $\dfrac{\sqrt{2}}{2}+i\dfrac{\sqrt{6}}{2}$

(c) $2\sqrt{2}+i2\sqrt{2}$

(d) -3

14.2

(a) $z=|z|\angle\theta$

(b) $z=2\angle\dfrac{\pi}{2}$

(c) $z=1\angle\dfrac{3}{2}\pi$

(d) $z=|zvert\angle\theta$

14.3

(a) $z_1+3z_2=2+i+3(1-i)=5-2i$

(b) $(z_1+z_2)^2=(2+i+1-i)^2=9$

(c) $2z_1-3z_2=2(2+i)-3(1-i)=1+5i$

(d) $z_1z_2=(2+i)(1-i)=3-i$

(e) $\dfrac{z_1}{z_1+z_2}=\dfrac{2+i}{2+i+1-i}=\dfrac{2}{3}+\dfrac{1}{3}i$

(f) $\dfrac{z_1}{z_2^2}=\dfrac{2+i}{(1-i)^2}=\dfrac{2+i}{-2i}=\dfrac{-1+2i}{2}=-\dfrac{1}{2}+i$

14.4

(a) $z^4=2^4(\cos 4\cdot\dfrac{\pi}{6}+i\sin 4\cdot\dfrac{\pi}{6})=16(-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i)=-8+8\sqrt{3}i$

(b) $z^3=2\sqrt{2}(\cos\dfrac{3}{4}\pi-i\sin\dfrac{3}{4}\pi)=-2-2i$

(c) $z^6=4^6(\cos 2\pi+i\sin 2\pi=4^6$

(d) $z^{10}=32(\cos\dfrac{5}{2}\pi+i\sin\dfrac{5}{2}\pi)=32i$

14.5

(a) $(\sqrt{3}-i)^{\frac{1}{3}}=2^{\frac{1}{3}}(\cos\frac{\pi}{18}-i\sin\frac{\pi}{18}),\ \ 2^{\frac{1}{3}}(\cos\frac{13}{18}\pi-i\sin\frac{13}{18}\pi),\ \ 2^{\frac{1}{3}}(\cos\frac{25}{18}\pi-i\sin\frac{25}{18}\pi)$

(b) $(1+i)^{\frac{1}{5}}=\sqrt{2^{\frac{1}{5}}}=\left[\cos\frac{1}{5}(\frac{\pi}{4}+2k\pi)+i\sin\frac{1}{5}(\frac{\pi}{4}+2k\pi)\right],\ \ k=0,1,2,3,4$

(c) $(2+2\sqrt{3}i)^{\frac{1}{4}}=4^{\frac{1}{4}}\left[\cos\frac{1}{4}(\frac{\pi}{3}+2\pi k)+i\sin\frac{1}{4}(\frac{\pi}{3}+2\pi k)\right],\ \ k=0,1,2,3$

(d) $(1-i)^{\frac{1}{4}}=\sqrt{2^{\frac{1}{5}}}\left[\cos\frac{1}{5}(\frac{\pi}{4}+2\pi k)-i\sin\frac{1}{5}(\frac{\pi}{4}+2\pi k)\right],\ \ k=0,1,2,3,4$

14.6

(a) $\lim_{z\to i}(2z^2-3z+1)=-1-3i$

(b) $\lim_{z\to i}\frac{z^4-1}{z-i}=\frac{1-1}{-2i}=0$

14.7

(a) $f'(z)=4z-3$

(b) $f'(z)=3z^2-6z+5$

(c) $f'(z)=\frac{-1+i}{(z+1)^2}$

(d) $f'(z)=\frac{(4z+3)(z^2+1)-(sz^2+3z-1)2z}{(z^2+1)^2}$

14.8

(a) 해석적이지 않다.

(b) 해석적이지 않다.

(c) $z=1$을 제외한 모든 점에서 해석적이다.

(d) (c)와 같은 방법 이용

(e) 해석적이다.

(f) 해석적이다.

14.9

(a) $\ln z=\ln 2+i(\pi\pm 2\pi n),n=0,1,2,....$

$\mathrm{Ln}z=\ln 2+i\pi$

(b) $\ln z=2\sqrt{2}+i(\frac{3}{4}\pi\pm 2\pi n),n=0,1,2,...$

$\mathrm{Ln}z=\ln 2\sqrt{2}+i\frac{3}{4}\pi$

(c) $\ln z = \ln 2 + i(\frac{2}{3}\pi \pm 2n\pi), n = 0, 1, 2, \ldots$

$\mathrm{Ln} z = \ln 2 + i\frac{2}{3}\pi$

(d) $\ln z = \ln\sqrt{2} + i(-\frac{\pi}{4} \pm 2n\pi), n = 0, 1, 2, \ldots$

$\mathrm{Ln} z = \ln\sqrt{2} - i\frac{\pi}{4}$

(e) $\ln z = \ln\sqrt{2} + i(\frac{\pi}{4} \pm 2n\pi), n = 0, 1, 2, \ldots$

$\mathrm{Ln} z = \ln\sqrt{2} + i\frac{\pi}{4}$

(f) $\ln z = \ln 2 + i(-\frac{\pi}{2} \pm 2n\pi), n = 0, 1, 2, \ldots$

$\mathrm{Ln} z = \ln 2 - i\frac{\pi}{2}$

14.10

(a) $\ln(z_1 z_2) = \ln 4\sqrt{2} + i(-\frac{\pi}{4} \pm 2n\pi), n = 1, 2, 3, \ldots$

$\mathrm{Ln}(z_1 z_2) = \ln 4\sqrt{2} - i\frac{\pi}{4}$

(b) $\ln(z_1 z_2) = \ln 2\sqrt{2} + i(-\frac{3}{4}\pi \pm 2n\pi), n = 0, 1, 2, \ldots$

$\mathrm{Ln}(z_1 z_2) = \ln 2\sqrt{2} - i\frac{3}{4}\pi$

14.11

(a) $-\frac{52}{3} - \frac{64}{3}i$

(b) $-\frac{4}{3} + \frac{4}{3}i$

(c) $-i\pi$

(d) $i\pi$

14.12

(a) $\int_C (z^2)dz = 0$

(b) $\int_C (2z^2 - 1)dz = 0$

14.13

(a) $\int_{C_1} f(z)\,dz = \frac{(1+i)^3}{3}$

(b) $\int_{C_2} f(z)\,dz = \frac{(1+i)^3}{3}$

(c) $\int_{C} f(z)\,dz = 0$

14.14

(a) $\int_{C} 2z\,dz = 2i-1$

(b) $\int_{C} 2z\,dz = 2i-1$

(c) $\int_{C} f(z)\,dz = 2i-1$

∴ (a), (b), (c) 모두 동일한 결과가 나온다.

14.15

(a) $\int_{C} (3z-1)dz = -\frac{11}{2}+4i$

(b) $\int_{C} e^z dz = e^{1+2i}-1$

14.16

$\int_{C} \frac{1}{z-a}\,dz = 2\pi i$

14.17

$\int_{C} \frac{f(z)}{z-a}\,dz = e^2(2\pi i)$

14.18

(a) $\int_{C} \frac{\cos z}{(z-2)^3}\,dz = -\pi i(\cos 2)$

(b) $\int_{C} \frac{\sin z}{z^3-3z^2+4}\,dz = 2\pi i(-\frac{1}{9}\sin 1-\frac{1}{9}\sin 2+\frac{1}{3}\cos 2)$

14.19

$$e^{-1/z^2} = \sum_{n=0}^{\infty} \frac{\left(-\frac{1}{z^2}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{z^{2n}} = 1 - \frac{1}{z^2} + \frac{1}{2!} \frac{1}{z^4} - + \dots$$

$$Res(e^{-\frac{1}{z^2}}, 0) = 0$$

14.20

$$f(z) = \frac{3}{z-2} + \frac{1}{6} - \frac{1}{6^2}(z-2) + \frac{1}{6^3}(z-2)^2 - + \dots, \quad 0 < |z-2| < 6$$

14.21

(a) $\int_C \frac{\sin z}{z^2} dz = 2\pi i$

(b) $\int_C \frac{\cos z}{z^3 + z^2} dz = 2\pi i(-1 + \cos 1)$