

CHAPTER 06 라플라스 변환

[6.1 라플라스 변환]

1.  $F(s) = e^{-s} \left( \frac{1}{s} + \frac{1}{s^2} \right)$

2.  $F(s) = \frac{s}{s^2 + 1} (e^{-\pi s} + 1)$

3.  $F(s) = -\frac{k}{s} (e^{-sa} - 1)$

4.  $F(s) = \frac{1}{s} (e^{-3s} - 2e^{-s} + 1)$

5.  $\frac{s}{s^2 + \pi^2}$

6.  $\frac{\omega \cos \theta + s \sin \theta}{s^2 + \omega^2}$

7.  $\frac{1}{2} \left( \frac{1}{s} - \frac{s}{s^2 + 4} \right)$

8.  $e^{-b} \frac{1}{s - a}$

9.  $\frac{1}{s^2 - 2s + 2}$

10.  $\frac{s + 1}{s^2 + 2s + 2}$

11. (a) 증명 생략  
(b) 증명 생략  
(c) 증명 생략

12. 증명 생략

13.  $\frac{\sqrt{\pi}}{\sqrt{s}}$

14.  $\frac{1}{2} \frac{\sqrt{\pi}}{s^{\frac{3}{2}}}$

CHAPTER 06 라플라스 변환

15.  $\frac{3\sqrt{\pi}}{4 \cdot s^{\frac{5}{2}}}$

16.  $\frac{1}{5!}t^5$

17.  $\frac{1}{2}e^{\frac{\sqrt{2}}{2}t}$

18.  $2\cos 2t$

19.  $\frac{1}{2}\sin 2t$

20.  $\cos \frac{1}{2}t$

21.  $\frac{1}{2}\cos \frac{1}{2}t$

22.  $\cosh 5t - \frac{6}{5}\sinh 5t$

23.  $\cos 6t - \frac{6}{5}\sin 5t$

24.  $-e^{-t} + e^{4t}$

25.  $\frac{1}{5}e^{4t} + \frac{1}{5}e^{-t}$

26.  $1 + 3\cos 3t - \sin 3t$

27.  $\frac{2}{13}e^{2t} - \frac{2}{13}\cos 3t + \frac{3}{13}\sin 3t$

28.  $y(t) = \frac{4}{3}\cos t + \sin t - \frac{1}{3}\cos 2t$

29.  $y(t) = 2e^t - e^{2t}$

CHAPTER 06 라플라스 변환

30.  $y(t) = \cos t + \frac{2}{3} \sin t - \frac{1}{3} \sin 2t$

31.  $y = \frac{5}{4}e^t + \frac{7}{24}e^{-t} - \frac{2}{3}e^{2t} + \frac{1}{8}e^{3t}$

32.  $y(t) = \frac{1}{2}(e^t - e^{-t})$

[6.2 라플라스 변환의 성질]

33.  $\frac{1}{(s-2)^2}$

34.  $\frac{2}{(s+2)^3}$

35.  $\frac{2}{(s-2)^3} + 2\frac{1}{(s-2)^2} + \frac{1}{s-2}$

36.  $\frac{2}{(s-2)^2 + 4}$

37.  $\frac{s-2}{(s-2)^2 + 4}$

38.  $\frac{1}{2}t^2e^{-t}$

39.  $\frac{1}{6}t^3e^t$

40.  $(1+3t)e^{-2t}$

41.  $(2\cos \sqrt{3}t - \frac{2}{\sqrt{3}} \sin \sqrt{3}t)e^t$

42.  $(7\cos t + 6\sin t)e^{-t}$

43.  $(t - t^2 + \frac{2}{3}t^3)e^t$

CHAPTER 06 라플라스 변환

44.  $\left(t - 2t^2 + \frac{2}{3}t^3\right)e^{-t}$

45.  $y(t) = (1 + 3t)e^{-2t}$

46.  $y = (2\cos \sqrt{3}t - \frac{2}{\sqrt{3}}\sin \sqrt{3}t)e^t$

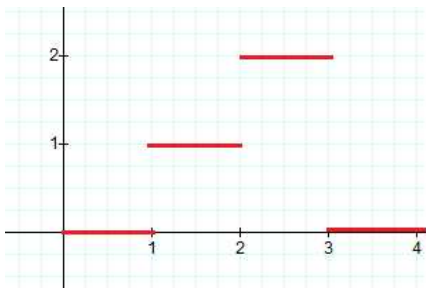
47.  $y(t) = (\frac{7}{5}\cos t + \frac{6}{5}\sin t)e^{-t} - \frac{2}{5}\cos t + \frac{1}{5}\sin t$

48.  $y(t) = (-\cos t + \sin t)e^{-t} + e^{-t}$

49.  $y(t) = -\frac{3}{4}e^t - \frac{1}{2}t^2e^t + \frac{1}{12}e^{-t} + \frac{2}{3}e^{2t}$

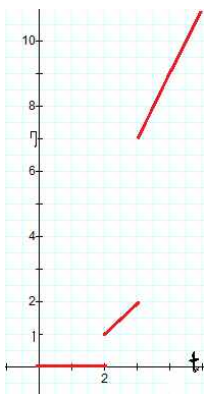
50.  $y(t) = (t - t^2 + \frac{2}{3}t^3)e^t$

51.



$$\begin{cases} 0, & 0 \leq t < 1 \\ 1, & 1 \leq t < 2 \\ 2, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

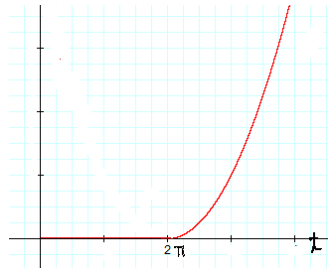
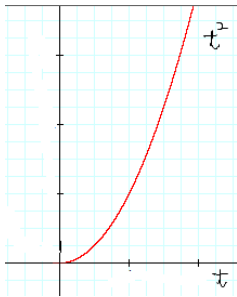
52.



$$\begin{cases} 0 & 0 \leq t < 2 \\ t-1 & 2 \leq t < 3 \\ 2t+1 & t \geq 3 \end{cases}$$

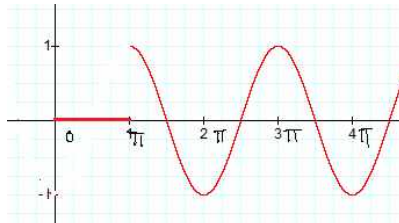
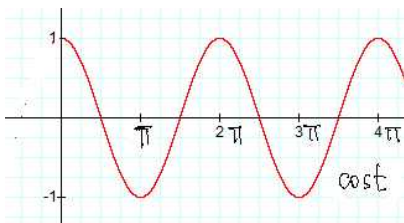
CHAPTER 06 라플라스 변환

53.



$$\begin{cases} 0 & 0 \leq t < 2\pi \\ f(t-2\pi) = (t-2\pi)^2 & t \geq 2\pi \end{cases}$$

54.



$$\begin{cases} 0 & 0 \leq t \leq \pi \\ f(t-\pi) = \cos(t-\pi) & t \geq \pi \\ = \cos(\pi-t) & \\ = -\cos t & \end{cases}$$

55.  $\frac{2}{s^3}e^{-s}$

56.  $(\frac{2}{s^3} - \frac{2}{s})e^{-2s}$

57.  $\frac{1}{s^2}e^{-\pi s} - \frac{1}{s^2}e^{-3\pi s} - \frac{2\pi}{s}e^{-3\pi s}$

58.  $\frac{1}{s}(e^{-s} + e^{-2s} - 2e^{-3s})$

59.  $(\frac{1}{s^2} + \frac{1}{s})e^{-2s} + (\frac{1}{s^2} + \frac{5}{s})e^{-3s}$

60.  $t^2e^{-2t}$

61.  $\frac{1}{\sqrt{3}}\sin\sqrt{3}(t-2)e^{-(t-2)}u(t-2)$

62.  $e^{t-1}\cos\sqrt{2}(t-1) \cdot u(t-1)$

CHAPTER 06 라플라스 변환

63.  $\frac{1}{2} \sinh 2t$

64.  $e^{3(t-2)} \cosh \sqrt{6}(t-2) u(t-2)$

65.  $e^{(t-1)} u(t-1) + e^{(t-2)} u(t-2) + e^{(t-3)} u(t-3) + e^{(t-4)} u(t-4)$

66.  $\left(1 - (t-3) + \frac{1}{2}(t-3)^2 - e^{-(t-3)}\right) u(t-3)$

67.  $y(t) = (1 - \cos t) - (1 - \cos(t-\pi)) u(t-\pi) = 1 - \cos t - (1 + \cos t) u(t-\pi)$

68.  $y(t) = (\cos t - \cos \sqrt{2} t) + (\cos t + \cos \sqrt{2}(t-\pi)) u(t-\pi)$

69.  $y(t) = e^{2t} - e^t + \left(\frac{1}{2} - e^{t-3} + \frac{1}{2} e^{2(t-3)}\right) u(t-3)$

70.  $y(t) = \cos t + 2t - 2 \sin t - 2[2t + 3 \sin t - \pi - 1 + (\pi - 1) \cos t] u(t-\pi)$

71.  $y(t) = \frac{1}{17} [\cos t + 4 \sin t - (\cos t + \frac{9}{2} \sin t) e^{-\frac{1}{2}t}] - \frac{1}{17} [\cos t + 4 \sin t - (\cos t + \frac{9}{2} \sin t) e^{-\frac{1}{2}(t-\pi)}] u(t-\pi)$

72.  $y(t) = [-1 + \frac{1}{2} \cos t(t-1) + \frac{1}{4} e^{-(t-1)} + \frac{1}{4} e^{(t-1)}] u(t-1) + [-1 + \frac{1}{2} \cos t(t-2) + \frac{1}{4} e^{-(t-2)} + \frac{1}{4} e^{(t-2)}] u(t-2)$

73.  $i(t) = -\frac{1}{10} + t + \frac{1}{10} e^{-10t} - \left(\frac{19}{10} + (t-2) - \frac{19}{10} e^{-10(t-2)}\right) u(t-2)$

[6.3 합성곱과 주기함수의 라플라스 변환]

74.  $\frac{4s}{(s^2 + 4)^2}$

75.  $\frac{-2(5s^2 - 1)}{(s^2 + 1)^3}$

76.  $\frac{s^2 - 2s}{(s^2 + 2s + 2)^2}$

CHAPTER 06 라플라스 변환

77.  $\frac{2s+2}{(s^2+2s+2)^2}$

78.  $y(t) = -\frac{1}{2}t \sin t - \frac{1}{2}(\sin t - t \cos t)$

79.  $y(t) = 2\cos 2t + \sin 2t + \frac{1}{4}t \sin 2t$

80.  $\frac{2}{s^3} \frac{1}{s^2+1}$

81.  $\frac{1}{s+1} \frac{s}{s^2+4}$

82.  $\frac{1}{s^2(s-1)}$

83.  $\frac{s}{(s^2+1)(s^2+1)}$

84.  $\frac{2}{s^3(s-1)^2} + \frac{2e^{-\pi s}}{s^2+4}$

85.  $\frac{2}{(s-1)^3} - \frac{2}{s^2}$

86.  $t + e^{-t} - 1$

87.  $\frac{1}{2}(-\cos t + \sin t + e^t)$

88.  $\frac{1}{2}te^t - \frac{1}{2}e^t + \frac{1}{2}\cos t$

89.  $\frac{1}{2}(t^2 - 2t - 2e^{-t} + 2)$

90.  $y(t) = \frac{2}{5}e^{-t}\cos t + \frac{1}{5}e^{-t}\sin t - \frac{2}{5}\cos t + \frac{1}{5}\sin t$

91.  $y(t) = -e^{-2t} + 12te^{-2t} + te^{-2t} * g(t)$

CHAPTER 06 라플라스 변환

92.  $y(t) = 1 + t + \frac{2}{3}e^t + \frac{4}{3}e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2}t$

93.  $y(t) = 2e^{-t}$

94.  $y(t) = 1 - \cos t + \sin t$

95.  $y(t) = -1 + e^{-t} + 2t^2e^{-t}$

96.  $y(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{\frac{1}{2}t} \cos t - \frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}e^{\frac{1}{2}t} \sin \frac{\sqrt{3}}{2}t$

97.  $i(t) = (1 - e^{-2(t-3)})u(t-3)$

98.  $i(t) = 100\left(\frac{1}{2} - t - \frac{1}{2} \cos \sqrt{2}t + \frac{1}{\sqrt{2}} \sin \sqrt{2}t\right) + 100\left(\frac{1}{2} + (t-1) - \frac{1}{2} \cos \sqrt{2}(t-1) - \frac{1}{\sqrt{2}} \sin \sqrt{2}(t-1)\right)u(t-1)$

99.  $\frac{1 - se^{-s} - e^{-s}}{s^2(1 - e^{-s})}$

100.  $\frac{1 + e^{-\pi s}}{(1 - e^{-\pi s})(1 + s^2)}$

101.  $\frac{1}{e^{-s} + 1} \left( \frac{e^{-s}}{s} + \frac{e^{-s} - 1}{s^2} \right)$

[6.4 라플라스 변환의 성질]

102.  $y(t) = \begin{cases} 0 & t < \pi \\ -\frac{1}{3} \sin 3t & t \geq \pi \end{cases}$

103.  $y(t) = \begin{cases} 0 & t < \pi \\ -\sin t & \pi \leq t < 2\pi \\ -2\sin t & t \geq 2\pi \end{cases}$

104.  $y(t) = \begin{cases} \cosh t & t < 1 \\ \cosh t + 10\sinh(t-1) & t \geq 1 \end{cases}$



CHAPTER 06 라플라스 변환

$$105. y(t) = \begin{cases} e^t - 1 & t < 2 \\ e^t + e^{t-2} - 2 & t \geq 2 \end{cases}$$

$$106. y(t) = -\frac{2}{5} \cos t + \frac{1}{5} \sin t + \left( \frac{2}{5} \cos t + \frac{1}{5} \sin t \right) e^{-t} - e^{-(t-\pi)} \sin t u(t-\pi)$$

$$107. y(t) = \frac{2}{\sqrt{31}} \sin \frac{\sqrt{31}}{4} \left( t - \frac{\pi}{2} \right) e^{-\frac{1}{4} \left( t - \frac{\pi}{2} \right)} u \left( t - \frac{\pi}{2} \right)$$

$$108. \begin{aligned} x_1 &= c_1 \cos 2t + c_2 \sin 2t \\ x_2 &= -c_1 \sin 2t + c_2 \cos 2t \end{aligned}$$

$$109. \begin{aligned} x_1 &= \left( c_1 \cos \frac{\sqrt{7}}{2} t + c_2 \sin \frac{\sqrt{7}}{2} t \right) e^{\frac{5}{2} t} \\ x_2 &= \left( -\left( \frac{3}{4} c_1 + \frac{\sqrt{7}}{4} c_2 \right) \cos \frac{\sqrt{7}}{2} t + \left( \frac{\sqrt{7}}{4} c_1 - \frac{3}{4} c_2 \right) \sin \frac{\sqrt{7}}{2} t \right) e^{\frac{5}{2} t} \end{aligned}$$

$$110. \begin{aligned} x_1 &= -\frac{1}{2} - \frac{13}{2} e^{2t} + 7(\cosh \sqrt{2} t) e^t + \frac{10}{\sqrt{2}} (\sinh \sqrt{2} t) e^t \\ x_2 &= -\frac{1}{2} + \frac{7}{2} e^{2t} - 3(\cosh \sqrt{2} t) e^t - \frac{4}{\sqrt{2}} (\sinh \sqrt{2} t) e^t \end{aligned}$$

$$111. \begin{aligned} x &= -\frac{1}{3} e^{-2t} + \frac{4}{3} e^t \\ y &= \frac{1}{3} e^{-2t} + \frac{2}{3} e^t \end{aligned}$$

$$112. \begin{aligned} y_1 &= 1 - \cos t + \sin t + \sin(t-2)u(t-2) + (1 - \cos(t-2))u(t-2) \\ y_2 &= (\cos t + \sin t - 1) + (\cos(t-2) + \sin(t-2) - 1)u(t-2) \end{aligned}$$

$$113. \begin{aligned} y_1 &= -2 \cos t + \frac{7}{2} \sin t - \frac{1}{2} t \cos t + e^t + e^{-t} \\ y_2 &= e^t + e^{-t} + 2 \cos t - \frac{7}{2} \sin t + \frac{1}{2} t \cos t \end{aligned}$$

$$114. \begin{aligned} x &= \frac{13}{2} + 5t + \frac{3}{4} t^2 - \frac{13}{2} e^t + \frac{5}{2} t e^t \\ y &= -\frac{1}{2} + \frac{1}{2} e^t - \frac{1}{2} t e^t \end{aligned}$$