

2.1 연습문제

1. 동차: $\alpha = 0$, 해가 없다;
 $\alpha = \pm 1$, 해가 무수히 많다.
2. $\alpha \approx 1$ 에서, 오차가 발생한다.
3. (a) 해가 없다.
(b) 해가 무수히 많다.
4.
$$\begin{cases} x_1 = -697.3 \\ x_2 = 343.9 \end{cases} \quad \begin{cases} x_1 = -720.79976 \\ x_2 = 356.28760 \end{cases}$$
5.
$$\mathbf{r} = \begin{bmatrix} -0.001343 \\ -0.001572 \end{bmatrix}, \quad \hat{\mathbf{r}} = \begin{bmatrix} -0.0000001 \\ 0.0000000 \end{bmatrix},$$

$$\mathbf{e} = \begin{bmatrix} -0.001 \\ -0.001 \end{bmatrix}, \quad \hat{\mathbf{e}} = \begin{bmatrix} -0.659 \\ 0.913 \end{bmatrix}$$
6. (a) $x_2 = 1, x_1 = 0$
(b) $x_2 = 1, x_1 = 1$
(c) $b_1 = b_2 = 1$ 로 두자. 그러면 정확한 해 $x_2 = 1, x_1 = 0$ 을 얻는다.
7. (a)
$$\begin{cases} x_1 = 1.6034 + 0.4165i \\ x_2 = -0.4793 - 1.5664i \\ x_3 = 3.2039 + 1.2425i \end{cases}$$

(b)
$$\begin{cases} x_1 = 1.7915 + 0.1034i \\ x_2 = 1.2743 - 0.9389i \\ x_3 = -1.0544 - 3.517i \end{cases}$$

(c) $x_1 \approx -7.233, \quad x_2 \approx 1.133,$
 $x_3 \approx 2.433, \quad x_4 = 4.5$

2.1 컴퓨터 연습문제

6. $\mathbf{z} = [2i, i, i, i]^T, \lambda = 1 + 5i;$
 $\mathbf{z} = [1, 2, 1, 1]^T, \lambda = 2 + 6i;$
 $\mathbf{z} = [-i, -i, 0, -i]^T, \lambda = -3 - 7i;$
 $\mathbf{z} = [1, 1, 1, 0]^T, \lambda = -4 - 8i$
7. (a) $(3.75, 90^\circ); \quad (3.27, -65.7^\circ); \quad (0.775, 172.9^\circ)$
(b) $(2.5, -90^\circ); \quad (2.08, 56.3^\circ); \quad (1.55, -60.2^\circ)$

2.2 연습문제

1.
$$\begin{bmatrix} 1/2 & 5/2 & -4 & -1 \\ 1/4 & -1/2 & -5/19 & -62/19 \\ 3/4 & 9/10 & 38/5 & 9/10 \\ 4 & 1 & 0 & 4 \end{bmatrix}$$

2. $\mathbf{x} = [1/3, 3, 1/3]^T$

3.
$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 \\ 3 & -3 & 0 & 6 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

5.
$$\begin{bmatrix} 1/4 & 5/2 & 7/4 & 1/2 \\ \underline{4} & \underline{2} & \underline{1} & \underline{2} \\ \underline{1/2} & \underline{0} & \underline{5/9} & \underline{17/9} \\ \underline{1/4} & \underline{3/5} & \underline{27/10} & \underline{1/5} \end{bmatrix}$$

6. $l = (1, 3, 2)$, 두 번째 피벗 행은 세 번째 행이다.

8. $x_3 = -1, x_2 = 1, x_1 = 0$

10. $x_4 = -1, x_3 = 0, x_2 = 2, x_1 = 1$

13. (b) $x_3 = 1, x_2 = 1, x_1 = 1$

(d) $x_1 \approx 4.267, x_2 \approx -4.133, x_3 \approx -2.467$

17. $n(n+1)$

18. $\left[\frac{29}{10}(n^2 - 1) + \frac{7}{30}n(n-1)(2n-1) \right] 10^{-6} \text{ 초}$

19.

n	10	10^2	10^3	10^4
시간	$\frac{1}{3} \times 10^{-3} \text{ sec.}$	$\frac{1}{3} \text{ sec.}$	5.56 min.	3.86 days
비용	0.005 cents	5 cents	\$46.30	\$46,296.30

21. $\mathbf{U}^T \mathbf{y} = \mathbf{b}, \mathbf{L}^T \mathbf{x} = \mathbf{y}$ 을 푼다.

23. (a) $x_1 = \frac{5}{9}, x_2 = \frac{2}{9}, x_3 = \frac{1}{9} \times 10^{-9}$

2.2 컴퓨터 연습문제

2. $[3.4606, 1.5610, -2.9342, -0.4301]^T$
3. $[6.7831, 3.5914, -6.4451, -1.5179]^T$
4. $2 \leq n \leq 10, x_i \approx 1$ for all i ; for large n , many $x_i \neq 1$
5. $b_i = n^2 + 2(i-1)$ 6. $x_2 = 1, \quad x_i = 0$ for $i \neq 2$

2.3 연습문제

2. (a) $5n-4$
3. $n+2nk-k(k+1)$
6. 그렇다.
7. $D^{-1}AD = \text{대각행렬 } [\pm\sqrt{a_{i-1}c_{i-1}}, d_i, \pm\sqrt{a_i c_i}]$

2.3 컴퓨터 연습문제

3.
$$\begin{cases} d_i \leftarrow d_i - 1/d_{i-1} \\ b_i \leftarrow b_i - b_{i-1}/d_{i-1} \quad (2 \leq i \leq n) \end{cases}$$
4.
$$\begin{cases} x_1 = 1 \\ x_i = 1 - (4x_{i-1})^{-1} \quad (2 \leq i \leq 100) \end{cases}$$
11. (a)
$$\begin{cases} x_1 \leftarrow b_1/a_{11} \\ x_i \leftarrow \left(b_i - \sum_{j=1}^{n-1} a_{ij}x_j \right) / a_{ii} \quad (2 \leq i \leq n) \end{cases}$$
12.
$$\begin{cases} c_i \leftarrow c_i/d_i \\ b_i \leftarrow b_i/d_i \\ d_{i+1} \leftarrow d_{i+1} - a_{i+1}c_i \\ b_{i+1} \leftarrow b_{i+1} - a_{i+1}b_i \quad (1 \leq i \leq n-1) \end{cases}$$
- $$\begin{cases} b_n \leftarrow b_n/d_n \\ b_i \leftarrow b_i - c_i b_{i+1} \quad (i = n-1, \dots, 1) \end{cases}$$