

MSE, 미적분학

[연습문제 답안 이용 안내]

- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

Chapter 05 연습문제 답안

《Section 5.2》

1. (a) $\int_{-1}^4 6dx = 5 \times 6 = 30$

(b) $\int_{-1}^3 xdx = \frac{9}{2} - \frac{1}{2} = 4$

(c) $\int_{-2}^2 x^3 dx = 0$

2. (a) $\int_1^5 \ln x dx$

(b) $-\int_{1/2}^1 \ln x dx$

(c) $\int_1^7 \ln x dx - \int_{1/3}^1 \ln x dx$

3. (a) $\int_1^3 x^2 dx$

(b) $\int_0^3 x^3 dx$

(c) $\int_{-2}^0 x^3 dx$

4. (a) 음수

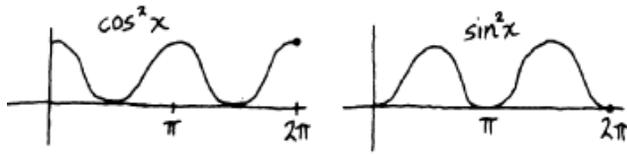
(b) 양수

5. (a) 참

(b) 거짓

(c) 참

6. (a)



$$(b) \int_0^{2\pi} \sin^2 x dx = \int_0^{2\pi} (1 - \cos^2 x) dx$$

$$\int_0^{2\pi} \sin^2 x dx = 2\pi - \int_0^{2\pi} \cos^2 x dx = 2 \int_0^{2\pi} \sin^2 x dx = 2\pi, \int_0^{2\pi} \sin^2 x dx = \pi$$

7. (a) A_4

$$(b) A_5 = \frac{1}{2} A_1$$

8. (a) 10

$$(b) \int_a^b 4x^3 dx = 10$$

$$4 \int_a^b x^3 dx = 10$$

$$\int_a^b x^3 dx = \frac{10}{4}$$

9.

$$x^2 + y^2 = R^2$$

$$y = \sqrt{R^2 - x^2}$$

$$A = \int_{-R}^R \sqrt{R^2 - x^2} dx$$

$$2 \int_{-R}^R \sqrt{R^2 - x^2} dx$$

《Section 5.3》

1. $(2x^3 - \frac{3}{2}x^2 + 2x)|_1^2 = 14 - (-\frac{11}{2}) = 39/2$
2. $(3t - \frac{1}{2}t^2)|_1^3 = \frac{9}{2} - \frac{5}{2} = 2$
3. $(\frac{1}{2}x^6 - \frac{2}{3}x^3)|_0^2 = \frac{80}{3} - 0 = \frac{80}{3}$
4. $-\frac{1}{2}\cos 2x|_{\pi/3}^{\pi/2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
5. $\tan^{-1}x|_0^1 = \frac{1}{4}\pi - 0 = \frac{1}{4}\pi$
6. $-(\frac{1}{\pi})\cos \pi x|_0^{1/2} = 0 - (-1/\pi) = 1/\pi$
7. $\ln x|_1^5 = \ln 5 - \ln 1 = \ln 5$
8. $-1/12x^2|_2^3 = -\frac{1}{108} - (-\frac{1}{48}) = \frac{5}{432}$
9. $2x^{3/2}|_1^5 = 2\sqrt{5^3} - 2 = 10\sqrt{5} - 2$
10. $-\frac{2}{3}(10-x)^{3/2}|_1^9 = -\frac{2}{3} - (-\frac{2}{3}\sqrt{9^3}) = -\frac{2}{3} + \frac{54}{3} = \frac{52}{3}$
11. $\frac{1}{2}\ln(2x+1)|_3^4 = \frac{1}{2}(\ln 9 - \ln 7)$
12. $4(5 - -2) = 28$
13. $\tan x|_0^{\pi/4} = 1 - 0 = 1$

14. $\int_2^5 1 dx = 1(5-2) = 3$

15. $\int_{-1}^2 (x^6 + 4x^3 + 4) dx = \left(\frac{1}{7}x^7 + x^4 + 4x\right)\Big|_{-1}^2 = \frac{296}{7} - \left(-\frac{22}{7}\right) = \frac{318}{7}$

16. $\frac{1}{4} \cdot \frac{1}{4} \left(\frac{1}{2}x + 7\right)^4 \cdot 2\Big|_2^4 = \frac{1}{8}(9^4 - 8^4) = \frac{2465}{8}$

17. $\frac{1}{8} \left(\frac{x+3}{5}\right)^8 \cdot 5\Big|_{-1}^1 = \frac{5}{8} \left[\left(\frac{4}{5}\right)^8 - \left(\frac{2}{5}\right)^8\right]$

18. $\frac{1}{3}(\ln 4 - \ln 5)$

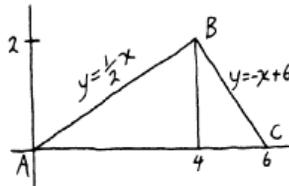
19. $(10/\pi) \sin \frac{1}{2} \pi x \Big|_{-3}^0 = 0 - 10/\pi = -10/\pi$

20. $-1/4(2x-9)^2 \Big|_2^3 = -\frac{1}{4} \left[\frac{1}{9} - \frac{1}{25}\right] = -\frac{4}{225}$

21. $\frac{1}{3} e^{3x} \Big|_0^1 = \frac{1}{3} e^3 - \frac{1}{3}$

22. (a) $\frac{1}{2} \times 6 \times 2 = 6$

(b) $\int_0^4 \frac{1}{2} x dx + \int_4^6 (-x+6) dx = 6$



23. $\frac{\int_0^\pi \sin x dx}{\pi - 0} = \frac{-\cos x \Big|_0^\pi}{\pi} = 2/\pi$

24. (a) $\frac{1}{4} x^4 + C$

(b) $15/4$

$$25. \quad \int_2^3 5dx + \int_3^4 0dx + \int_4^6 x^3 dx = 5(3-2) + 0 + \frac{1}{4}x^4 \Big|_4^6 = 5 + 260 = 265$$

26.

$$\int_3^{10} |4-x| dx = \int_3^4 (4-x) dx + \int_4^{10} (x-4) dx = \left(4x - \frac{1}{2}x^2\right) \Big|_3^4 + \left(\frac{1}{2}x^2 - 4x\right) \Big|_4^{10} = \frac{1}{2} + 18 = \frac{37}{2}$$

$$27. \quad (a) \quad \int_0^2 (x^3 - 6x^2 + 8x) dx - \int_2^4 (x^3 - 6x^2 + 8x) dx = 8$$

$$(b) \quad \int_0^5 \sqrt{9-x} dx - 10 = \frac{8}{3}$$

《Section 5.4》

1. (a) $h = \frac{1}{4}, x_0 = 0, y_0 = f(x_0) = 1$
 $x_1 = \frac{1}{4}, y_1 = f(x_1) = 1.0019512;$
 $x_2 = \frac{1}{2}, y_2 = f(x_2) = 1.0307764;$
 $x_3 = \frac{3}{4}, y_3 = f(x_3) = 1.1493475;$
 $x_4 = 1, y_4 = f(x_4) = 1.4142136;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4) = 1.0894134$
- (b) $h = \frac{1}{6}, x_0 = 0, y_0 = f(x_0) = 1$
 $x_1 = \frac{1}{6}, y_1 = f(x_1) = 0.027399;$
 $x_2 = \frac{1}{3}, y_2 = f(x_2) = 0.1053605;$
 $x_3 = \frac{1}{2}, y_3 = f(x_3) = 0.2231436;$
 $x_4 = \frac{2}{3}, y_4 = f(x_4) = 0.3677248;$
 $x_5 = \frac{5}{6}, y_5 = f(x_5) = 0.5273579;$
 $x_6 = 1, y_6 = f(x_6) = 0.6931472;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6) = 0.2639393$
- (c) $h = \frac{1}{8}, x_0 = 0, y_0 = f(x_0) = 1$
 $x_1 = \frac{9}{8}, y_1 = f(x_1) = 0.4125705;$
 $x_2 = \frac{5}{4}, y_2 = f(x_2) = 0.3386243;$
 $x_3 = \frac{11}{8}, y_3 = f(x_3) = 0.2778079;$
 $x_4 = \frac{3}{2}, y_4 = f(x_4) = 0.2285714;$
 $x_5 = \frac{13}{8}, y_5 = f(x_5) = 0.1889996;$
 $x_6 = \frac{7}{4}, y_6 = f(x_6) = 0.1572482;$
 $x_7 = \frac{15}{8}, y_7 = f(x_7) = 0.1317211;$
 $x_8 = 2, y_8 = f(x_8) = 0.1111111;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + y_8) = 0.2543498$

(d) $h = \frac{1}{6}, x_0 = 0, y_0 = f(x_0) = 1$
 $x_1 = \frac{1}{6}, y_1 = f(x_1) = 0.9992287;$
 $x_2 = \frac{1}{3}, y_2 = f(x_2) = 0.9877302;$
 $x_3 = \frac{1}{2}, y_3 = f(x_3) = 0.9394131;$
 $x_4 = \frac{2}{3}, y_4 = f(x_4) = 0.8207548;$
 $x_5 = \frac{5}{6}, y_5 = f(x_5) = 0.6173908;$
 $x_6 = 1, y_6 = f(x_6) = 0.3678794;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6) = 0.8449433$

2. $h = \frac{1}{4}, x_0 = 1, y_0 = f(x_0) = 1$
 $x_1 = \frac{5}{4}, y_1 = f(x_1) = 0.64;$
 $x_2 = \frac{3}{2}, y_2 = f(x_2) = \frac{4}{9};$
 $x_3 = \frac{7}{4}, y_3 = f(x_3) = 0.3265306;$
 $x_4 = 2, y_4 = f(x_4) = 0.25;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4) = 0.5004176$
 $-(1/x)_1^2 = -\frac{1}{2} + 1 = 0.5$

《Section 5.6》

1. $-1/4x^4|_3^\infty = 0 + 1/4(81) = 1/324$

2. $\frac{5}{6}x^{6/5}|_2^\infty = \infty - \frac{5}{6} \cdot 2\sqrt[5]{2} = \infty$

3. $-1/2x^2|_\infty^{-2} = -\frac{1}{8} + 0 = -1/8$

4. $-1/x|_{-1}^0 = -(1/0-) - 1 = \infty$

5. $\ln x|_{0+}^2 = \ln 2 - (-\infty) = \infty$

6. $\int_{-2}^{0-} 1/x^3 dx + \int_{0+}^3 1/x^3 dx = -1/2x^2|_{-2}^0 - 1/2x^2|_{0+}^3$
 $= -(1/0+) + \frac{1}{8} - \frac{1}{18} + \frac{1}{0+} = -\infty + \infty$
 답 없음

7. $\tan^{-1} x|_\pi^0 = 0 - (-\pi/2) = \pi/2$

8. $\frac{1}{2}e^{4x}|_{-\infty}^0 = \frac{1}{2} - 0 = \frac{1}{2}$

9. $\int_2^{4-} + \int_{4+}^5 = -\frac{3}{2}(4-x)^{2/3}|_2^{4-} - \frac{3}{2}(4-x)^{2/3}|_{4+}^5$
 $= \frac{3}{2}\sqrt[3]{4} - \frac{3}{2}$

10. $\int_{-2}^{0-} + \int_{0+}^3 = -1/x|_{-2}^0 - 1/x|_{0+}^3 = \infty + \infty = \infty$

11. $\ln x|_{0+}^\infty = \infty - \infty = \text{발산}$

12. 발산

13. $\int_{-\infty}^0 e^x dx + \int_0^\infty e^x dx = e^x|_{-\infty}^0 + e^x|_0^\infty = 1 - 0 - 0 + 1 = 2$

14. $-\ln \cos x \Big|_0^{\pi/2} = -\ln(0+) + \ln 1 = \infty + 0 = \infty$

15. $F(\infty) - F(-\infty) = \frac{1}{2} \left\{ 0 + \frac{\pi}{2} - \left(0 - \frac{\pi}{2} \right) \right\} = \pi/2$

16. $(x \ln x - x) \Big|_{0+}^1 = -1$

《복습문제》

1. (a) $\frac{1}{7}x^7|_{-1}^1 = \frac{1}{7} - \left(-\frac{1}{7}\right) = \frac{2}{7}$
- (b) $\int_{-1}^{0-} + \int_{0+}^1 = -1/5x^5|_{-1}^{0-} - 1/5x^5|_{0+}^1 = \infty + \infty = \infty$
- (c) $-1/5x^5|_1^\infty = 0 + \frac{1}{5} = \frac{1}{5}$
- (d) $\left(\frac{1}{3}x^3 + 3x\right)|_1^2 = \frac{26}{3} - \frac{10}{3} = \frac{16}{3}$
- (e) $\frac{1}{3} \cdot \frac{2}{3}(3x+4)^{3/2}|_1^2 = \frac{2}{9}(10\sqrt{10} - 7\sqrt{7})$
- (f) $-\frac{1}{3}e^{-3x}|_2^\infty = \frac{1}{3}e^{-6}$
- (g) $2\sin\frac{1}{2}x|_0^\pi = 0$
- (h) $3(7-4) = 9$
- (i) $\int_{-1}^0 e^x dx + \int_0^3 e^{-x} dx = e^x|_{-1}^0 - e^{-x}|_0^3$
 $= 1 - e^{-1} - e^{-3}$
- (j) $\frac{1}{4} \cdot \frac{1}{6}(2x+5)^6 \cdot \frac{1}{2}|_{-1}^0 = \frac{1}{48}[5^6 - 3^6] = \frac{14896}{48} = 310\frac{1}{3}$
- (k) $-4\ln(2-x)|_0^1 = 4\ln 2$
- (l) $17 - 15 = 2$
2. (a) $\frac{3}{2} + 1 + 2 - 2 - 12 = -\frac{19}{2}$
- (b) $\int_0^6 f(x) dx = \int_0^1 (3x+1) dx + \int_1^3 (-4x+8) dx + \int_3^6 (-4) dx$
 $= \left(\frac{3}{2}x^2 + x\right)|_0^1 + (-2x^2 + 8x)|_1^3 - 4(3) = \frac{5}{2} + 0 - 12 = -\frac{19}{2}$

3. $h = \frac{1}{6}, x_0 = 0, y_0 = f(x_0) = 1$
 $x_1 = \frac{1}{6}, y_1 = f(x_1) = 1.0068733;$
 $x_2 = \frac{1}{3}, y_2 = f(x_2) = 1.0266901;$
 $x_3 = \frac{1}{2}, y_3 = f(x_3) = 1.0573713;$
 $x_4 = \frac{2}{3}, y_4 = f(x_4) = 1.0962894;$
 $x_5 = \frac{5}{6}, y_5 = f(x_5) = 1.1409243;$
 $x_6 = 1, y_6 = f(x_6) = 1.1892071;$
 $\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6) = 1.069769$

4. $I = II$

5. $\left(\int_1^e 1/x dx\right)/(e-1) = \frac{1}{e-1}$

6. $A = -\int_{-1}^1 [-(x^3 - 2x^2 - 5x + 6)] dx + \int_1^3 [-(x^3 - 2x^2 - 5x + 6)] dx$
 $= \left(\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x\right)\Big|_{-1}^1 - \left(\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{5}{2}x^2 + 6x\right)\Big|_1^3 = \frac{32}{3} + \frac{16}{3} = 16$

7. (a) 0

(b) $\int_{-3}^3 f(x) dx = 2 \int_0^3 f(x) dx$