최종수정 : 2021. 04. 21

<수식이 보이는 공학수학> (개정판)

연습문제 답안

Chapter_01

1.1 $x=4$

1.2 $a=-2$이면, $0=0$이므로 부정

$a\neq -2$이면, $x=\dfrac{1}{a+3}$

1.3 $y=1$

1.4 $a=0,\ b\neq 0$이면 $x=-\dfrac{c}{b}$

$a=0,\ b=0$이고 $c=0$이면 부정, $c\neq 0$이면 불능

$a\neq 0$이면 $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

1.5 $x=-\dfrac{4}{3}$, $x=2$

1.6 $\lambda=-1$, $\lambda=8$

1.7 $t=1$, $t=-\dfrac{2}{3}$

1.8 $x^2-4x+1=0$

1.9 $x^2-2\alpha x+\alpha^2+\beta=0$

1.10 $x=1$, $x=\dfrac{-1\pm\sqrt{3}\,i}{2}$

1.11 $x=1$, $x=\pm i$

1.12 $y=\pm 2i$, $y=\pm 1$

1.13 $t=-2\pm\sqrt{2}\,i$, $t=-4\pm\sqrt{10}$

1.14 $f'(x)=5x^4-12x^2+4x$

1.15 $f'(t)=\dfrac{-t^2-6t-1}{(t^2+t+2)^2}$

1.16 $f'(x)=-e^{-3x}(3\cos 2x+2\sin 2x)$

1.17 $f(x)=4(2x^2-x+1)^3(4x-1)$

1.18 $f'(x)=4\omega\sin^3\omega x\cos\omega x$

1.19 $f'(x)=\dfrac{4x+3}{2x^2+3x+7}$

1.20 $f'(x)=ab\,e^{\sin bx}\cos bx$

1.21 $f'(t)=\dfrac{2t\cos 2t-\sin 2t}{t^2}$

1.22 $\displaystyle\int f(x)dx=\frac{2}{5}(1-x)^2\sqrt{1-x}-\frac{2}{3}(1-x)\sqrt{1-x}+c$

1.23 $\displaystyle\int f(t)dt=\frac{1}{2}t\sin 2t+\frac{1}{4}cos2t+c$

1.24 $\displaystyle\int f(x)dx=\frac{1}{2}x^2\ln 2x-\frac{1}{4}x^2+c$

1.25 $\displaystyle\int f(\lambda)d\lambda=32\lambda\sin\frac{\lambda}{4}-4(\lambda^2-32)\cos\frac{\lambda}{4}+c$

1.26 $\int f(y)dy = \frac{1}{20}(y^4+2)^5 + c$

1.27 $\int f(t)dt = -\frac{1}{3}(1-t^2)\sqrt{1-t^2} + c$

1.28 $\int f(x)dx = -e^{-2x^2} + c$

1.29 $\int f(y)dy = \frac{1}{7}\sin^7 y + c$

1.30 $x = 0.2527$

1.31 $x = 1.8235$

1.32 $x = -1.3258$

1.33 생략

1.34 생략

1.35 생략

1.36 생략

1.37 $\frac{1}{2}\sin\alpha + \frac{\sqrt{3}}{2}\cos\alpha = \sin\left(\alpha + \frac{\pi}{3}\right)$

1.38 $3\sqrt{3}\sin\beta - 3\cos\beta = \sin\left(\beta - \frac{\pi}{6}\right)$

1.39 $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$

Chapter_02

2.1 3계, 선형

2.2 2계, 비선형

2.3 2계, 선형

2.4 2계, 비선형

2.5 $y(x) = -x\cos x + \sin x + \pi$

2.6 $y(x) = \frac{1}{2}\sin 2x + 5$

2.7 $y(x) = (2x+1)\sin x + (2-x)\cos x - 2$

2.8 $y = ce^{-x}$

2.9 $\frac{x^2}{16} + \frac{y^2}{25} = c$

2.10 $y(x) = \frac{1}{\ln\left|\frac{1}{cx}\right|}$

2.11 $y(x) = ce^{\frac{1}{2n}x^2}$

2.12 $4y^2 + 9x^2 = 13$

2.13 $i(t) = Ce^{-\frac{B}{A}t}$ (2021-04-21 업데이트)

2.14 $y(x) = \sqrt{2x - \sin 2x + 4}$

2.15 $x = e^{-0.000121 \cdot 3000}$, 약 69.6%

2.16 $e^x \sin 2y = c$

2.17 $y \ln|x| = c$

2.18 $y \sin x - yx + 3y = c$ (2021-04-21 업데이트)

2.19 $y = 4x^{-x}$

2.20 $x(e^y + 3) + y(x+2) + e^x(x-1) = -1$

2.21 $3y\{y(6\ln x + 1) + 16\} + 2(x+1) = 16$

2.22 $y(x) = \frac{5}{8}e^{2x} + ce^{-2x}$

2.23 $y(x) = -\frac{6}{5}e^{3x} + ce^{8x}$

2.24 $y(x) = -\frac{3}{8} + ce^{-8x}$

2.25 $y(x) = \frac{1}{5}e^{2x}(3\cos 3x - \sin 3x) + ce^{-4x}$

2.26 $y(x) = 2x^2 + x$

2.27 $y = -2\cos x(\cos x - 2)$

2.28 $y^3 = 2 + ce^{-\frac{3}{2}x^2}$

2.29 $y = \dfrac{1}{cxe^{-3x} - x/3}$

2.30 $y^4 = \dfrac{1}{ce^x - 3x - 5}$

2.31 (a) $Q(t) = Q_0 e^{\frac{1}{R}\left(\frac{1}{R} - \frac{1}{C}\right)t}$

(b) $t = \dfrac{4.6R^2C}{C - R}$

2.32 (a) $I(t) = \dfrac{E_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$, $Q(t) = \dfrac{E_0}{R}t + \dfrac{E_0 L}{R^2}e^{-\frac{R}{L}t}$

(b) $I(t) = ce^{-\frac{R}{L}t} + \dfrac{E_0}{R^2 + \omega^2 L^2}(R\sin\omega t - \omega L\cos\omega t)$

$$Q(t) = \int I(t)dt = -\frac{E_0}{R^2 + w^2L^2}\left(\frac{R}{w}\cos wt + L\sin wt\right) + \frac{cL}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

2.33 (a) $I(t) = ce^{-\frac{t}{RC}}$

(b) $I(t) = ce^{-\frac{1}{RC}t} + \dfrac{\omega E_0 C}{1 + (\omega RC)^2}(\cos\omega t - \omega RC\sin\omega t)$

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Chapter_03

3.1 $y = c_1 e^{-8x} + c_2 e^{3x}$

3.2 $y = c_1 e^{-4x} + c_2 e^{3x}$

3.3 $y = (c_1 + c_2 x)e^{4x}$

3.4 $y = e^{-2x}(A\cos\sqrt{11}\,x + B\sin\sqrt{11}\,x)$

3.5 $y = e^{-2x}(A\cos 3x + B\sin 3x)$

3.6 $y = c_1 e^{-1.2x} + c_2 e^{-0.7x}$

3.7 $y'' - \frac{7}{2}y' + 3y = 0$ 또는 $2y'' - 7y' + 6y = 0$

3.8 $y'' - 6y' + 8y = 0$

3.9 $y'' - 14y' + 49y = 0$

3.10 $y'' - 2\sqrt{17}\,y' + 17y = 0$

3.11 $y'' + 25y = 0$

3.12 $y'' + 2\gamma y' + (\gamma^2 + \delta^2)y = 0$

3.13 $y = e^{-4x} + 6e^{2x}$

3.14 $y = (1 + x)e^{3x}$

3.15 $y = 2e^{-0.3x}\sin 2x$

3.16 $y = (2 + 9x)e^{-3.5x}$ (2021-04-05 업데이트)

3.17 $y = e^{-x}\left(\cos\omega x + \frac{1}{\omega}\sin\omega x\right)$

3.18 $y = e^{-3x}\sin 0.7x$

3.19 $y = 3e^{\frac{7}{3}x} + e^{-3x}$ (2021-04-05 업데이트)

3.20 $y = (1 + 10x)e^{-8x}$ (2020-12-08 업데이트)

3.21 $y = c_1 x^5 + c_2 x^{-3}$

3.22 $y = c_1 x^{2+\sqrt{3}} + c_2 x^{2-\sqrt{3}}$ (2021-04-05 업데이트)

3.23 $y = (c_1 + c_2 \ln x)x^2$

3.24 $y = (c_1 + c_2 \ln x)x^{-2.5}$

3.25 $y = x^{-2}[A\cos(2\ln x) + B\sin(2\ln x)]$

3.26 $y = \sqrt{x}\,[A\cos(3\ln x) + B\sin(3\ln x)]$

3.27 $y = (3 - 9\ln x)x^3$

3.28 $y = 10x^{0.25} + x^{-5}$

3.29 $y = 3\cos(4\ln x)$

3.30 $y = (c_1 + c_2 \ln x)x^{\frac{1-a}{2}}$

3.31 $y = y_h + y_p = c_1 e^{3x} + c_2 e^{-2x} - 0.14\cos x - 0.02\sin x$

3.32 $y = A\cos 2x + B\sin 2x + x^2 + e^{-2x}$

3.33 $y = y_h + y_p = e^{-2x}(A cos\sqrt{2}x + B sin\sqrt{2}x) + \frac{8}{7}e^{-3x}$

3.34 $y = y_h + y_p = c_1 e^{4x} + c_2 e^{-x} + \frac{1}{5}xe^{4x}$

3.35 $y = e^{-x}\cos 10x + 0.1e^{x}$

3.36 $y = \left(\frac{1}{2}x^2 - 1\right)e^{-x}$

3.37 $y = e^{0.5x} - 2\cos x + e^x$

3.38 (a) 생략

(b) $y(t) = A\cos\sqrt{\frac{k}{m}}t + B\sin\sqrt{\frac{k}{m}}t$

3.39 (a) $my'' + cy' + ky = 0$

(b) $y_p(t) = \frac{F_0(k - m\omega^2)}{(k - m\omega^2)^2 + \omega^2 c^2}\cos\omega t + \frac{F_0\omega c}{(k - m\omega^2)^2 + \omega^2 c^2}\sin\omega t$

3.40 $I(t) = e^{-4t}(1.5\cos 3t - \frac{10}{3}\sin 3t) - 1.5\cos 10t + 1.6\sin 10t$

Chapter_04

4.1 $y''' - 6y'' + 11y' - 6y = 0$

4.2 $y''' + 3y'' - 4y = 0$

4.3 $y^{(4)} + 5y'' + 4y = 0$

4.4 $y''' - 6y'' + 12y' - 8y = 0$

4.5 $y^{(4)} - 6y''' + 11y'' - 6y' = 0$

4.6 $y''' + y'' + y' + y = 0$

4.7 $y^{\mathrm{iv}} + 2y'' + y = 0$

4.8 $y''' + 5y'' + 3y' - 9y = 0$

4.9 $y = -1 - \frac{1}{2}e^x + \frac{1}{6}e^{-x} + \frac{1}{3}e^{2x}$

4.10 $y = e^x + 3\cos 10x + \sin 10x$

4.11 $y = -\cos x + 2\sin x - x\cos x - \frac{1}{2}x\sin x$

4.12 $y = \frac{1}{9}e^x - \frac{1}{3}\left(\frac{1}{3} + x\right)e^{-2x}$

4.13 7계 미분방정식

$y = c_1 e^{ax} + (c_2 + c_3 x)e^{bx} + e^{px}(A_1\cos qx + B_1\sin qx) + xe^{px}(A_2\cos qx + B_2\sin qx)$

4.14 $y = A\cos x + B\sin x + Ce^{-x}$

4.15 $y = C_1 e^x + C_2 xe^x + C_3 x^2 e^x$

4.16 $y = (C_1 + C_2 x + C_3 x^2)e^{4x}$

4.17 $y = C_1 e^{2x} + C_2 e^{-2x} + A\cos\sqrt{2}x + + B\sin\sqrt{2}x$

4.18 $y=(A_1+A_2x)\cos 3x+(B_1+B_2x)\sin 3x$

4.19 $y=(A_1+A_2x)\cos 2\pi x+(B_1+B_2x)\sin 2\pi x$

4.20 $y=4e^{-x}+e^{\frac{1}{2}x}\left(2\cos\frac{\sqrt{3}}{2}x+4\sqrt{3}\sin\frac{\sqrt{3}}{2}x\right)$ (2021-04-21 업데이트)

4.21 $y=e^{x}+3\cos 10x+\sin 10x$

4.22 $y=\frac{5}{4}e^{2x}+\frac{3}{4}e^{-2x}-2\cos 2x+\frac{1}{2}\sin 2x$

4.23 $y=c_1e^{5x}+c_2e^{-5x}+A\cos 5x+B\sin 5x-\frac{1}{16}\sinh x$

4.24 $y=y_h+y_p=c_1e^{2x}+c_2e^{-2x}+A\cos 2x+B\sin 2x+\frac{1}{2x}$

4.25 $y=c_1e^{1.5x}+c_2e^{-1.5x}+A\cos 2x+B\sin 2x-0.014\sin \pi x+\frac{1}{14}e^{2x}$

4,26 $y=3+(2-x)e^{-x}+x^2$

4.27 $y=(3-25x^2)e^{-x}+5x^3e^{-x}$

4.28 $y=3e^{2x}-4e^{-2x}+2\cos x$

Chapter_05

5.1 $\lambda_1=2,\ \lambda_2=-2,\ \mathbf{x}^{(1)}=\begin{bmatrix}1\\1\end{bmatrix},\ \boldsymbol{x}^{(2)}=\begin{bmatrix}1\\-1\end{bmatrix}$

5.2 $\lambda_1=-2,\ \lambda_2=-4,\ \mathbf{x}^{(1)}=\begin{bmatrix}1\\1\end{bmatrix},\ \mathbf{x}^{(2)}=\begin{bmatrix}1\\-1\end{bmatrix}$

5.3 $\lambda_1=0,\ \lambda_2=-3a,\ \mathbf{x}^{(1)}=\begin{bmatrix}1\\2\end{bmatrix},\ \mathbf{x}^{(2)}=\begin{bmatrix}1\\-1\end{bmatrix}$

5.4 $\lambda_1=0,\ \lambda_2=-0.036,\ \mathbf{x}^{(1)}=\begin{bmatrix}1\\5\end{bmatrix},\ \mathbf{x}^{(2)}=\begin{bmatrix}1\\-1\end{bmatrix}$

5.5 $\lambda_1=1,\ \lambda_2=-3,\ \mathbf{x}^{(1)}=\begin{bmatrix}1\\1\end{bmatrix},\ \mathbf{x}^{(2)}=\begin{bmatrix}5\\1\end{bmatrix}$

5.6 $\mathbf{y}=c_1\begin{bmatrix}1\\1\end{bmatrix}e^{2t}+c_2\begin{bmatrix}1\\-1\end{bmatrix}e^{-2t}$

5.7 $\mathbf{y}=c_1\begin{bmatrix}1\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\3\end{bmatrix}e^{-t}$

5.8 $\mathbf{y}=c_1\begin{bmatrix}0.75\\1\end{bmatrix}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{t}$

5.9 $\mathbf{y}=c_1\begin{bmatrix}-3\\1\end{bmatrix}e^{3t}+c_2\begin{bmatrix}3\\1\end{bmatrix}e^{9t}$

5.10 $y_1=\frac{1}{2}(e^{t}-e^{-t})$ or $\sinh t$

$y_2=\frac{1}{2}(e^{t}+e^{-t})$ or $\cosh t$

5.11 $y_1=-5e^{-3t}+5e^{t}$

$y_2=-e^{-3t}+5e^{t}$

5.12 $\begin{cases} y_1 = 4.5\cos 2t - 0.8\sin 2t \\ y_2 = -0.8\cos 2t - 4.5\sin 2t \end{cases}$

5.13 $y_1 = 3 + 9e^{2t}$
$y_2 = -1 + 3e^{2t}$

5.14 $y_1 = -\cos t - \sin t + c_1$
$y_2 = \cos t - \sin t + c_2$

5.15 $y_1 = e^{-t} + c_1$
$y_2 = -e^{t} + c_2$

5.16 $y_1 = c_1 e^{-4t} - 2c_2 e^{t} + \frac{5}{2} + 13e^{2t}$
$y_2 = 2c_1 e^{-4t} + c_2 e^{t} + 3 + 7e^{2t}$

5.17 $y_1 = c_1 e^{t} + 3c_2 e^{2t} - \frac{5}{2}t - \frac{13}{4}$
$y_2 = -c_1 e^{t} - 2c_2 e^{2t} + 3t + \frac{7}{2}$

5.18 $y_1 = c_1 e^{t} + 5c_2 e^{-3t} - 2$
$y_2 = c_1 e^{t} + c_2 e^{-3t} - 4$

5.19 $y_1 = C_1 e^{-9t} + C_2 e^{-3t} + 12t + 31$
$y_2 = 3C_1 e^{-9t} - 3C_2 e^{-4t} - 72t - 36$

5.20 $y_1 = 2e^{-t} + t^2$
$y_2 = -e^{-t} - t$

5.21 $y_1 = \cos 3t - \sin 3t + 0.5$
$y_2 = \cos 3t + \sin 3t - 2$

5.22 $y_1 = \frac{39}{5}e^{-2t} - \frac{9}{5}e^{3t} + 2t^2 - 4t - 6$
$y_2 = \frac{39}{5}e^{-2t} - \frac{9}{5}e^{3t} + 3t^2 - 4t - 15$

5.23 $y_1 = -\frac{5}{3}e^{2t} + e^{5t}$
$y_2 = -\frac{2}{3}e^{2t} + e^{5t}$

5.24 $y_1 = 16\cos 2t - 6\sin 2t + e^{t} - 16e^{-t}$
$y_2 = -3\cos 2t - 8\sin 2t - e^{t} + 4e^{-t}$

5.25 $y_1 = 16e^{10t} - e^{-4t} + 2\sin 2t$
$y_2 = 12e^{10t} + e^{-4t} + \sin 2t$

5.26 $I_1 = -8e^{-2t} + 5e^{-0.8t} + 3$
$I_2 = -4e^{-2t} + 4e^{-0.8t}$

Chapter_06

6.1 $y = a_0 \sum_{m=0}^{\infty} \frac{(-2x)^m}{m!}$

6.2 $y = a_0 \sum_{m=0}^{\infty} \frac{(3x)^m}{m!}$

6.3 $y = a_0\left(1 + x^2 + \frac{1}{2!}x^4 + \frac{1}{3!}x^6 + \cdots + \frac{1}{m!}x^{2m} + \cdots\right)$

6.4 $y = \frac{a_0 + a_1}{2}\left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots\right) + \frac{a_0 - a_1}{2}\left(1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \cdots\right)$

6.5 $y = a_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{2^k k!} x^{2k} + a_1 \sum_{k=0}^{\infty} \frac{(-1)^k 2^k k!}{(2k+1)!} x^{2k+1}$

6.6 $y = a_0\left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \frac{1}{12960}x^9 - \cdots\right) + a_1\left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7 - \frac{1}{45360}x^{10} - \cdots\right)$

6.7 $y = a_0 \sum_{m=0}^{\infty} \frac{x^m}{m!}$

6.8 $y \approx 2 - x + x^2 + \frac{1}{4}x^4 + \frac{3}{20}x^5$

6.9 (a) $y = a_0(1 - x + x^2 - x^3 + \cdots)$

(b) $y = x^2 - x + a_0(1 - x + x^2 - x^3 + \cdots)$

6.10 $y = a_0 \sum_{m=0}^{\infty} x^m$

6.11 $y = a_0(1 + x)$

6.12 $y = a_0\left(1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \cdots\right)$

6.13 $a_2 = -\frac{n(n+1)}{2!}a_0$

6.14 $a_3 = -\frac{(n-1)(n+2)}{3!}a_1$

6.15 $a_{s+2} = -\frac{(n-s)(n+s+1)}{(s+2)(s+1)}a_s$

6.16 $y_1(x) = 1 - \frac{n(n+1)}{2!}x^2 + \frac{(n-2)n(n+1)(n+3)}{4!}x^4 - \cdots$

$y_2(x) = x - \frac{(n-1)(n+2)}{3!}x^3 + \frac{(n-3)(n-1)(n+2)(n+4)}{4!}x^5 - \cdots$

6.17 $y_1(x) = a_0(x+1)$

$y_2(x) = A_0\sqrt{x}$

6.18 $y_1(x) = a_0 x \sum_{m=0}^{\infty} \frac{3 \cdot 2^{m+1}(m+1)^2}{(2m+3)!} x^m$

$y_2(x) = A_0 x^{-\frac{1}{2}} \sum_{m=0}^{\infty} \frac{1}{2^m m!} x^m$

6.19 $y_1 = \frac{\sin x}{x}$

6.20 $y_1(x) = a_0\left(1 - \frac{(2x)^2}{3!} + \frac{(2x)^4}{5!} - \frac{(2x)^6}{7!} + \cdots\right)$

6.21 $y_1(x) = a_0 x^3$

6.22 $y_1(x) = x^2$

6.23 $y_1(x) = a_0\sqrt{x}\left(1 - \frac{1}{3!}x^2 + \frac{1}{5!}x^4 - \frac{1}{7!}x^6 + \cdots\right)$

6.24 $y_1(x) = x$

Chapter_07

7.1 $F(s) = \frac{6A}{s^4} + \frac{2B}{s^3} + \frac{C}{s^2}$

7.2 $F(s) = \frac{20}{s^2 + 0.4^2}$

7.3 $F(s) = \frac{8}{s(s+4)}$

7.4 $F(s) = \frac{\sin 0.7s + \cos 0.7}{s^2 + 1}$

7.5 $F(s) = 5\delta(t - \pi)$

7.6 $F(s) = \frac{2(e^{-s} - e^{-2s})}{s}$

7.7 $F(s) = \frac{2\pi}{(s^2 + \frac{\pi^2}{4})}(1 - e^{-2s})$

7.8 $f(t) = 2\cos 2t - \frac{1}{4} sin2t$

7.9 $f(t) = (t^2 + \frac{2}{3})^2$

7.10 $f(t) = 2e^{3t} - 3e^{-t}$

7.11 $f(t) = 2e^{2t} + e^{3t}$

7.12 $f(t) = (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}})(e^{\sqrt{3}t} + e^{\sqrt{2}t})$

7.13 $f(t) = e^{-4\pi t}\sinh \pi t$

7.14 증명 생략

7.15 $F(s) = \frac{s^2 - 16}{(s^2 + 16)^2}$

7.16 $F(s) = \frac{s^2 + 2\pi^2}{s(s^2 + 4\pi^2)}$

7.17 $F(s) = \frac{1}{(s+3)^2}$

7.18 $F(s)=\dfrac{2}{s(s^2-4)}$

7.19 증명 생략

7.20 $f(t)=1-e^{-t}$

7.21 $f(t)=-t-\sinh t$

7.22 $f(t)=-2e^{-t}-t+2$

7.23 $f(t)=\dfrac{t}{a}+\dfrac{e^{-at}}{a^2}-\dfrac{1}{a^2}$

7.24 $F(s)=\dfrac{14}{(s+2)^4}$

7.25 $F(s)=\dfrac{2}{(s+5)^2+4}$

7.26 $F(s)=\dfrac{-s+5}{(s+4)^2+9}$

7.27 $f(t)=3(6t+1)e^{6t}$

7.28 $f(t)=e^{3t}\cos 3t$

7.29 $f(t)=2e^{-\omega t}\cosh 4t$

7.30 $y=-1+e^t$

7.31 $y=-\dfrac{1}{2}-\dfrac{1}{6}e^{-2t}+\dfrac{5}{3}e^t$

7.32 $y=e^{2t}-\dfrac{1}{4}t+\dfrac{1}{8}\sinh 2t$

7.33 $y=6t^2-5t+\dfrac{19}{12}$

7.34 $f(t)=\begin{cases} e^{-t}\cos t & (0<t<2\pi) \\ e^{-t}(\cos t+e^{2\pi}\sin t) & (t>2\pi) \end{cases}$

7.35 $f(t)=\begin{cases} \cos 3t+\dfrac{2}{3}sin3t & (0<t<\pi, t>2\pi) \\ \cos 3t+\dfrac{2}{3}sin3t+\dfrac{1}{3}(\sin 3(t-\pi)-\sin 3(t-2\pi)) & (\pi<t<2\pi) \end{cases}$

7.36 $0<t<2\pi$일 때 $i(t)=-e^{-t}(18\cos 3t+\dfrac{22}{3}\sin 3t)+18\cos t+4\sin t$

$t>2\pi$일 때 $i(t)=-e^{-t}(18\cos 3t+\dfrac{22}{3}\sin 3t)+18\cos t+4\sin t$ (2020-12-08 업데이트)

$+u(t-2\pi)\left\{e^{-(t-2\pi)}(18\cos 3t+\dfrac{22}{3}\sin 3t)\right\}$

$+u(t-2\pi)\{-18\cos t-4\sin t\}$

7.37 $F(s)=e^{-s}\cdot\dfrac{1}{s^2}-\dfrac{6e^{-s}}{s}$

7.38 $F(s)=e^{-2s}\left(\dfrac{6}{s^4}+\dfrac{12}{s^3}+\dfrac{12}{s^2}+\dfrac{8}{s}\right)$

7.39 $F(s)=\dfrac{se^{-\frac{\pi}{2}s}}{s^2+1}$

7.40 $F(s)=\dfrac{1}{s}-\dfrac{1}{s+4}-e^{-\pi s}\left\{\dfrac{1}{s}-\dfrac{1}{(s+4)e^{4\pi}}\right\}$

7.41 $f(t)=\dfrac{-e^{-at}+e^{-bt}}{a-b}$

7.42 $f(t)=\dfrac{-t\cos wt+\sin wt}{2w^2}$

7.43 $f(t)=\dfrac{1-\cos at}{a^2}$

7.44 $f(t)=3t^2e^{0.5t}$ $\quad\therefore\quad f(t)=t\cos t$

Chapter_08

8.1 $\begin{bmatrix}2&9\\4&2\end{bmatrix},\begin{bmatrix}2&9\\4&2\end{bmatrix}$

8.2 $\begin{bmatrix}2&-1\\-6&8\end{bmatrix},\begin{bmatrix}-2&1\\6&-8\end{bmatrix}$

8.3 $\begin{bmatrix}6&2\\-13&21\end{bmatrix},\begin{bmatrix}12&-16\\-46&54\end{bmatrix}$

8.4 $\begin{bmatrix}0&-5&4\\4&8&-2\end{bmatrix}$, 정의되지 않음

8.5 $\begin{bmatrix}15&-16&-1\\8&-5&-4\end{bmatrix}$, 정의되지 않음

8.6 증명 생략

8.7 증명 생략

8.8 증명 생략

8.9 $\begin{bmatrix}0&4&1\\0&16&4\\0&0&0\end{bmatrix}$,16

8.10 정의되지 않음, $\begin{bmatrix}10&1\\-17&-3\\19&1\end{bmatrix}$

8.11 정의되지 않음, $[-11\ \ -1]$

8.12 정의되지 않음, $[-4\ \ 12\ \ 7]$

8.13 $[-45\ \ -11]$

8.14 $\begin{bmatrix}-4&12&7\\-16&48&28\\0&0&0\end{bmatrix}$

8.15 $\begin{bmatrix}44\\176\\0\end{bmatrix}$

8.16 $17, \begin{bmatrix} 1 & 4 & 0 \\ 4 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8.17 $\begin{bmatrix} 5 & -8 & 4 \\ -8 & 11 & -5 \\ 4 & -5 & 25 \end{bmatrix}$

8.18 $\begin{bmatrix} 41 & -67 & 77 \\ -30 & 47 & -57 \\ 9 & -13 & 18 \end{bmatrix}$

8.19 $x=1, y=-1, z=2$

8.20 $x=1-\frac{7}{5}z,\ y=-\frac{3}{5}z$, z는 임의의 수

8.21 해가 존재하지 않는다.

8.22 $a=3c, b=2c+1, d=0, c$는 임의의 수

8.23 1

8.24 23

8.25 1

8.26 $(a-b)(b-c)(c-a)$

8.27 -22

8.28 $-(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

8.29 $4x^2y^2z^2$

8.30 증명 생략

8.31 증명 생략

8.32 $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

8.33 $\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

8.34 $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

8.35 $\begin{bmatrix} 0.3 & 0 & 0.1 \\ 0 & 0.5 & 0 \\ -0.1 & 0.1 & -0.1 \end{bmatrix}$

8.36 $\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

8.37 $\begin{bmatrix} 5 & -1 & 5 \\ 0 & 2 & 0 \\ -5 & 3 & -15 \end{bmatrix}$

Chapter_09

9.1 $\mathbf{a}=[1, -2, 2]$, $|\mathbf{a}|=3$

9.2 $\mathbf{a}=[0, -1, 0]$, $|\mathbf{a}|=1$

9.3 $\mathbf{a}=[-2, 4, 2]$, $|\mathbf{a}|=\sqrt{24}$

9.4 $\mathbf{a}=[\sqrt{5}, 3, -4]$, $|\mathbf{a}|=\sqrt{30}$

9.5 $\mathbf{a}=[-2c, -2b, 0]$, $|\mathbf{a}|=2\sqrt{c^2+b^2}$

9.6 $[5, 1, 2]$, $[5, 1, 2]$

9.7 $[32, -32, 14]$, $[32, -32, 14]$

9.8 $[18, -19, 7]$, $[-18, 19, -7]$

9.9 $\overrightarrow{AB}=\mathbf{u}(\mathbf{a}\cdot\mathbf{u})$, $\overrightarrow{BC}=\mathbf{a}-\mathbf{u}(\mathbf{a}\cdot\mathbf{u})$

9.10 $16, -19, -20$

9.11 $121, -110$

9.12 $36, -1$

9.13 $[-6, -4, 11]$, $[-20, 4, -18]$, $[-6, -4, 3]$

9.14 $[14, -8, -3]$, $[-12, 12, 3]$

9.15 $-52, -52$

9.16 $\mathbf{r}'(1)=2i+2j$, $\mathbf{u}'(1)=\dfrac{2i+2j}{\sqrt{8}}$

9.17 $\mathbf{r}'(2)=3i-k$, $\mathbf{u}'(2)=\dfrac{3i-k}{\sqrt{10}}$

9.18 $\mathbf{r}'((2n-1)\pi)=-j$, $\mathbf{u}'((2n-1)\pi)=-j$

9.19 $\mathbf{r}'(1)=i+2j+3k$, $\mathbf{u}'(1)=\dfrac{i+2j+3k}{\sqrt{14}}$

9.20 $\mathbf{r}'(\pi/2)=-3j+4k$, $\mathbf{u}'(1)=\dfrac{-3j+4k}{5}$

9.21 $\sinh 2$

9.22 $2\pi^2$

9.23 6

9.24 $\nabla f=2xy\mathbf{i}+x^2\mathbf{j}$

9.25 $\nabla f=2x\mathbf{i}+2y\mathbf{j}+2z\mathbf{k}$

9.26 $\nabla f=\dfrac{1}{2\sqrt{x+y}}\mathbf{i}+\dfrac{1}{2\sqrt{x+y}}\mathbf{j}$

9.27 $\nabla f=e^x\sin y\,\mathbf{i}+e^x\cos y\,\mathbf{j}$

9.28 $2(x+y+z)(\mathbf{i}+\mathbf{j}+\mathbf{k})$

9.29 $\dfrac{4}{5}\mathbf{i}+\dfrac{3}{5}\mathbf{j}$

9.30 $\dfrac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j})$

9.31 $\dfrac{1}{\sqrt{2}}(i+j)$

9.32 6
9.33 $2e$
9.34 0
9.35 $-2k$
9.36 xk
9.37 0

Chapter_10

10.1 $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = -\frac{1}{6}$

10.2 $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = -\frac{8}{3}$

10.3 (a) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = 4$

(b) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = \frac{6}{5}$

10.4 $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = 0$

10.5 (a) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = 34$, (b) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = \frac{70}{3}$

10.6 (a) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = -\frac{4}{15}$, (b) $\int_C \mathrm{F}(\mathrm{r}) \cdot d\mathrm{r} = \frac{1}{3}$

10.7 $\frac{7}{12}$

10.8 54

10.9 $\frac{\pi}{8}$

10.10 $\frac{8\pi}{3}(1+\pi^2)$

10.11 -6π

10.12 $-\frac{\pi}{2}$

10.13 0

10.14 $\frac{\pi^2}{8}(\cosh 2 - 1) - 2 \approx 1.408$

10.15 $-\frac{1}{3}$

10.16 $\frac{1}{12}$

10.17 1

10.18 24π

10.19 4

10.20 $\dfrac{abc^2}{8}$

10.21 3

10.22 $a^2(2e^a - 1)$

10.23 $-\dfrac{1}{6}$

10.24 $8\pi^2$

10.25 $\dfrac{64\pi}{3}$

10.26 ± 2

10.27 $\pm(1-\cos 1)$ (2020-12-08 업데이트)

10.28 $\pm\dfrac{3\pi}{2}$

10.29 0

10.30 $\pm 2(1-e^2)$ (2020-12-08 업데이트)

10.31 증명 생략, $\oint_C \mathrm{F} \cdot d\mathrm{r} = \pm\dfrac{1}{2}$

Chapter_11

11.1 그림 생략

11.2 그림 생략

11.3 그림 생략

11.4 그림 생략

11.5 그림 생략

11.6 증명 생략

11.7 증명 생략

11.8 증명 생략

11.9 증명 생략

11.10 $f(x) = \dfrac{\pi}{2} + \sum_{n=1}^{\infty}\left\{\dfrac{2(1-(-1)^n)}{n^2\pi}\cos nx\right\}$

11.11 $f(x) = \sum_{n=1}^{\infty}\dfrac{2}{n\pi}\{1-(-1)^n\}\sin nx$

11.12 $f(x) = \dfrac{\pi^2}{3} + \sum_{n=1}^{\infty}\left(\dfrac{4(-1)^n}{n^2}\cos nx\right)$

11.13 $f(x) = \dfrac{4}{3}\pi^2 + \sum_{n=1}^{\infty}\left(\dfrac{4}{n^2}\cos nx - \dfrac{4\pi}{n}\sin nx\right)$

11.14 $f(x)=\frac{1}{2}cosx+\frac{2}{\pi}\left(\frac{1}{1\cdot 3}sin2x+\frac{1}{3\cdot 5}sin4x+\cdots\right)$

11.15 $f(x)=1+\frac{4}{\pi}(\sin x+\frac{1}{3}\sin 3x+\cdots)$

11.16 $f(x)=\frac{4k}{\pi}(\sin\frac{\pi}{5}x+\frac{1}{3}\sin\frac{3\pi}{5}x+\cdots)$

11.17 $f(x)=\frac{1}{3}+\frac{2}{\pi}\left(\sin\frac{\pi}{3}\cos\frac{\pi}{3}x+\frac{1}{2}\sin\frac{2\pi}{3}\cos\frac{2\pi}{3}x+\frac{1}{4}\sin\frac{4\pi}{3}\cos\frac{4\pi}{3}x+\cdots\right)$

11.18 $f(x)=\frac{5}{4}-\frac{2}{\pi^2}\left(\cos\pi x+\frac{1}{9}\cos 3\pi x+\cdots\right)-\frac{1}{\pi}(3\sin\pi x+\frac{1}{2}\sin 2\pi x+\frac{3}{3}\sin 3\pi x+\cdots)$

11.19 $f(x)=\frac{4}{\pi}\left(\sin\frac{\pi}{2}x+\frac{1}{2}\sin\frac{2\pi}{2}x+\frac{1}{3}\sin\frac{3\pi}{2}x+\cdots\right)$

11.20 우함수

11.21 속하지 않음

11.22 우함수

11.23 기함수

11.24 속하지 않음

11.25 우함수

11.26 기함수

11.27 속하지 않음

11.28 Fcs : $f(x)=1$

Fss : $f(x)=\frac{4}{\pi}\left(\sin\frac{\pi}{2}x+\frac{1}{3}\sin\frac{3\pi}{2}x+\frac{1}{5}\sin\frac{5\pi}{2}x+\cdots\right)$

11.29 Fcs : $f(x)=1-\frac{8}{\pi^2}\left(\cos\frac{\pi}{2}x+\frac{1}{3^2}\cos\frac{3\pi}{2}x+\frac{1}{5^2}\cos\frac{5\pi}{2}x+\cdots\right)$

Fss : $f(x)=\frac{4}{\pi}\left(\sin\frac{\pi}{2}x-\frac{1}{2}\sin\frac{2\pi}{2}x+\frac{1}{3}\sin\frac{3\pi}{2}x-\cdots\right)$

11.30

Fcs : $f(x)=\frac{1}{3}+\frac{4}{\pi^2}\left(-\cos\pi x+\frac{1}{2^2}\cos 2\pi x-\frac{1}{3^2}\cos 3\pi x+\cdots\right)$

Fss : $f(x)=-2\left\{\left(\frac{4}{\pi^2}-\frac{1}{\pi}\right)\sin\pi x+\frac{1}{2\pi}\sin 2\pi x+\left(\frac{4}{3^3\pi^3}-\frac{1}{3\pi}\right)\sin 3\pi x+\frac{1}{4\pi}\sin 4\pi x+\cdots\right\}$

11.31 Fcs : $f(x)=\frac{e^2-1}{2}+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2+4}\{(-1)^n e^2-1\}\cos\frac{n\pi}{2}x$

Fss : $f(x)=\sum_{n=1}^{\infty}\frac{2n\pi}{n^2\pi^2+4}\{1-(-1)^n e^2\}\sin\frac{n\pi}{2}x$

11.32 Fcs : $f(x)=2-\frac{64}{\pi^2}\left(\frac{1}{2^2}\cos\frac{2\pi}{4}x+\frac{1}{6^2}\cos\frac{6\pi}{4}x+\frac{1}{10^2}\cos\frac{10\pi}{4}x+\cdots\right)$

Fss : $f(x)=\frac{32}{\pi^2}\left(\sin\frac{\pi}{4}x-\frac{1}{3^2}\sin\frac{3\pi}{4}x+\frac{1}{5^2}\sin\frac{5\pi}{4}x-\cdots\right)$

11.33 Fcs : $f(x)=\frac{3}{2}+\frac{12}{\pi^2}\left(\cos\frac{\pi}{3}x+\frac{1}{3^2}\cos\frac{3\pi}{3}x+\frac{1}{5^2}\cos\frac{5\pi}{3}x+\cdots\right)$

Fss : $f(x)=\frac{6}{\pi}\left(\sin\frac{\pi}{3}x+\frac{1}{2}\sin\frac{2\pi}{3}x+\frac{1}{3}\sin\frac{3\pi}{3}x+\cdots\right)$

Chapter_12

12.1 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{\cos\omega x\sin\omega}{\omega}d\omega$

12.2 $f(x)=\frac{1}{\pi}\int_0^\infty \left[\left\{\frac{a\sin\omega a}{\omega}+\left(\frac{\cos\omega a-1}{\omega^2}\right)\right\}\cos\omega x+\left\{-\frac{a\cos\omega a}{\omega}+\frac{\sin\omega a}{\omega^2}\right\}\sin\omega x\right]d\omega$

12.3 $f(x)=\frac{1}{\pi}\int_0^\infty \left\{\left(\frac{1-\cos\omega k}{\omega^2}\right)\cos\omega x+\frac{1}{\omega}\left(k-\frac{\sin\omega k}{\omega}\right)\sin\omega x\right\}d\omega$

12.4 증명 생략

12.5 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{\cos\omega-1+\omega\sin\omega}{\omega^2}\cos x\omega d\omega$ (푸리에 코사인 적분)

$f(x)=\frac{2}{\pi}\int_0^\infty \frac{\sin\omega-\omega\cos\omega}{\omega^2}\sin x\omega\, d\omega$ (푸리에 사인 적분)

12.6 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{\omega^2\sin\omega+2\omega\cos\omega-2\sin\omega}{\omega^3}\cos x\omega d\omega$ (푸리에 코사인 적분)

$f(x)=-\frac{2}{\pi}\int_0^\infty \frac{\omega^2\cos\omega-2\omega\sin\omega-2\cos\omega+2}{\omega^3}\cos x\omega d\omega$ (푸리에 사인 적분)

12.7 $f(x)=\frac{2a}{\pi}\int_0^\infty \frac{\cos\omega x}{\omega^2+a^2}d\omega$ (푸리에 코사인 적분)

$f(x)=\frac{2}{\pi}\int_0^\infty \frac{\omega\sin\omega x}{\omega^2+a^2}d\omega$ (푸리에 사인 적분)

12.8 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{1-e^{-1}(\cos\omega-\omega\sin\omega)}{1+\omega^2}\cos x\omega d\omega$ (푸리에 코사인 적분)

$f(x)=\frac{2}{\pi}\int_0^\infty \frac{e^{-1}(-\sin\omega-\omega\cos\omega)+\omega}{1+\omega^2}\sin x\omega d\omega$ (푸리에 사인 적분)

12.9 $f(x)=\frac{1}{2\pi i}\int_{-\infty}^\infty \frac{1}{\omega}\left\{1-\frac{1}{i\omega}(1-e^{-ik\omega})\right\}e^{ix\omega}d\omega$

12.10 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{1+\cos\pi\omega}{1-\omega^2}\cos x\omega d\omega$ (푸리에 코사인 적분)

12.11 $f(x)=\frac{2}{\pi}\int_0^\infty \frac{\omega(1+\cos\pi\omega)}{\omega^2-1}\sin x\omega d\omega$ (푸리에 사인 적분)

12.12 $F(\omega)=\frac{1}{\sqrt{2\pi}(1+i\omega)}$

12.13 $F(\omega)=\sqrt{\frac{\pi}{k}}e^{-\frac{\omega^2}{4k}}$

12.14 $F(\omega)=\frac{(1+i\omega)e^{-i\omega}-1}{\omega^2\sqrt{2\pi}}$

12.15 $F(\omega)=\frac{2}{\omega^2}\left(\frac{2\sin\omega}{\omega}-e^{i\omega}-1\right)$

12.16 (a) $F_c(\omega)=\sqrt{\frac{2}{\pi}}\frac{1}{\omega^2+1}$

(b) $F_s(\omega)=\sqrt{\frac{2}{\pi}}\frac{\omega}{\omega^2+\pi^2}$

12.17 (a) $F_c(\omega)=\sqrt{\frac{2}{\pi}}\left\{\frac{2\omega\cos\omega+(\omega^2-2)\sin\omega}{\omega^3}\right\}$

(b) $F_s(\omega)=\sqrt{\frac{2}{\pi}}\left\{\frac{2\omega\sin\omega+(2-\omega^2)\cos\omega-2}{\omega^3}\right\}$

Chapter_13

13.1 $u(x,y)=A(y)e^x+B(y)e^{-x}$

13.2 $u(x,y)=f(x)e^{-y}+g(y)$

13.3 $u(x,y)=c_1(x)\cos 2y+c_2(x)\sin 2y$

13.4 $u(x,y)=c_1(y)+c_2(y)x$

13.5 $u(x,y)=c(y)e^{-x^2}$

13.6 $u(x,y)=c_1(x)+c_2(x)e^{-3y}$

13.7 증명 생략

13.8 (a) 증명 생략

(b) $u=2\sin(y-3x)$

13.9 (a) $u=\frac{1}{6}x^3y^2+h(y)+g(x)$

(b) $u=\frac{1}{6}x^3y^2+\cos y-\frac{1}{6}y^2+x^2-1$

13.10 $u(x,t)=a\cos 3\pi t\sin 3\pi x$

13.11 $u(x,t)=\frac{4}{\pi^2}\left(\sin\pi x\cos\pi t-\frac{1}{3^2}\sin 3\pi x\cos 3\pi t+\cdots\right)$

13.12 $u(x,t)=\frac{8a}{\pi^2}\left(\sin\pi x\cos\pi t+\frac{1}{27}\sin 3\pi x\cos 3\pi t+\frac{1}{125}\sin 5\pi x\cos 5\pi t+\cdots\right)$

13.13 $u(x,t)=\frac{8}{\pi^2}\left(\frac{1}{4}\sin 2\pi x\cos 2\pi t-\frac{1}{36}\sin 6\pi x\cos 6\pi t+\frac{1}{100}\sin 10\pi x\cos 10\pi t+\cdots\right)$

13.14 $u(x,t)=\frac{4}{\pi^3}\left\{(4-\pi)\cos\pi t\sin\pi x+\cos 2\pi t\sin 2\pi x+\frac{4+3\pi}{27}\cos 3\pi t\sin 3\pi x+\cdots\right\}$

13.15 $G''+\lambda^2G=0$
$H''+k^2H=0$
$Q''+q^2Q=0$

Chapter_14

14.1 25

14.2 $13+9i$

14.3 13

14.4 -15

14.5 $-\frac{3}{5}(1+2i)$

14.6 $-297-54i$

14.7 4

14.8 $2-2i=2\sqrt{2}\left\{\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right\}$

14.9 $i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

$-i=\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)$

14.10 $\frac{1+i}{1-i}=\frac{(1+i)^2}{2}=i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

14.11 $1+\frac{\pi}{2}i=\sqrt{1+\frac{\pi^2}{4}}\left(\cos\tan^{-1}\frac{\pi}{2}+i\sin\tan^{-1}\frac{\pi}{2}\right)$

14.12 $1+\sqrt{3}i=2(\cos 60^\circ+i\sin 60^\circ)$

14.13 $\pi+\pi i=\pi\sqrt{2}(\cos 45^\circ+i\sin 45^\circ)$

14.14 $w_0=1$, $w_1=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$, $w_2=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

14.15 $w_k=\cos\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)+i\sin\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right) \quad (k=0,1,2)$

14.16 증명 생략

14.17 증명 생략

14.18 증명 생략

14.19 비해석적

14.20 해석적

14.21 비해석적

14.22 $z\neq 0$이면 해석적

14.23 증명 생략

14.24 증명 생략